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Batik Motif From The Movement Of Dynamics Harmoic Waves

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ABSTRACT

This batik motif is inspired by the movement of waves in the context of dynamic harmony. Through mathematical and philosophical analysis, wave motion is explored as a symbol of life that constantly changes yet remains harmonious. The methods used include studies on wave displacement, wave partitioning, and the dynamics of waves on a string fixed at one end. Each phase of wave movement is interpreted from the perspective of culture and life values, inviting the viewer to understand the beauty in change and appreciate diversity. Through this motif, the creator wishes to convey that beauty can be found in the dynamic fluctuations of life, inspiring one to face challenges with balance.

Introduction

Batik is a cultural heritage of Indonesia and a fusion of art and technology. Wave Batik is a batik motif derived from wave equations. Dynamic Harmony reflects balance in an ever-changing environment. This term represents the ability to maintain harmony despite the changes in life. These life changes are represented through motifs of wave movement, wave displacement, and wave partitioning.

Method

This research is an exploratory study employing an artistic qualitative approach, integrating the principles of harmonic wave physics into the creation process of batik motifs. Data were obtained through a literature review on dynamic waves, including rope waves, waves on wires with both ends fixed, as well as the concepts of wave deviation and separation. These physical principles were then visualized into motif designs. The creation process involved drawing mathematical representations of wave motion using design software, which were subsequently translated into batik motifs through manual and digital techniques. Data analysis was conducted qualitatively by observing visual forms, pattern regularity, and artistic harmony. Conclusions were drawn based on the successful integration of wave concepts into visual motifs that exhibit both aesthetic and scientific values.

Result and Discussion

1. Waves of a String with One End Tied

In Figure 1, a string with one end tied is vibrated. The string is given an initial displacement $f(x) = \sin(x)$ and an initial transverse velocity $g(x) = 0$. The special solution for the string displacement is

$$u(x,t) = \mathcal{L}^{-1}\left\{\left[-\frac{C}{2s}\int\left(\frac{1}{C^2}g(x) - \frac{s}{C^2}f(x)\right)e^{\frac{s}{C}x}dx\right]e^{-\frac{s}{C}x}dx + \left[\frac{C}{2s}\int\left(\frac{1}{C^2}g(x) - \frac{s}{C^2}f(x)\right)e^{-\frac{s}{C}x}dx\right]e^{\frac{s}{C}x}dx\right\}. \quad (1)$$

By substituting the given initial values, the solution is obtained.

$$u(x,t) = \mathcal{L}^{-1}\left\{\frac{s\sin(x)}{C^2+s^2}\right\} = \cos(Ct)\sin(x). \quad (2)$$

If we pay attention to all that intersect at a point x and intersect the x-axis at the same point. From the calculation, the intersection points are $x = 0$; $x = \pi$ and $x = 2\pi$. If the interval is extended, there will be other intersection points with the x-axis at points that are multiples of $x = \pi$. This happens because the solution to the wave equation forms a sine function that will always intersect the x-axis at points $x = n\pi, n = 0, \pm1, \pm2, \dots$

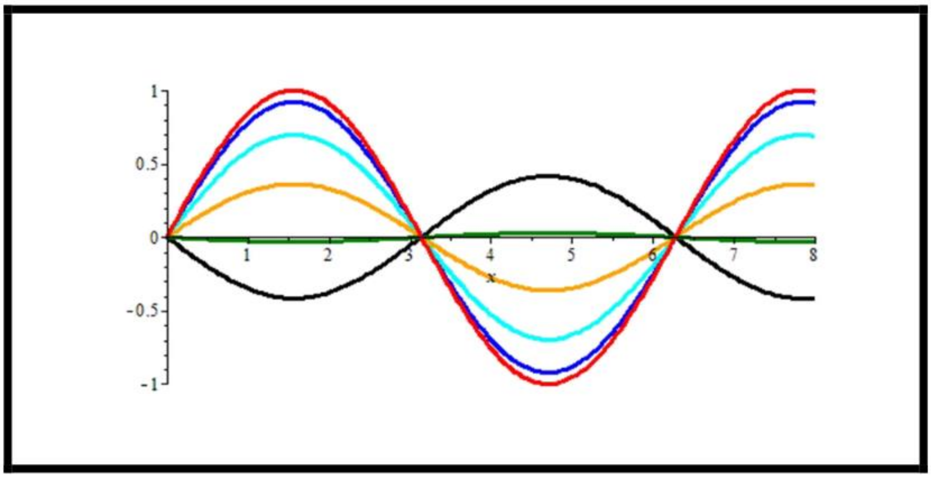

Figure 1. Wave Movement at Various Times $t = 0$ to $t = 3$ [1]

a. The Philosophical Meaning of a String Wave with One End Tied

The movement of waves reflects deep symbolism. The movement of waves contains deep

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philosophical meanings. Waves can symbolize various concepts such as the fluctuating life, the inevitability of change, or harmony in the changes of nature. In addition, waves are often considered a symbol of continuous change in life, because waves always move forward without stopping. The motif of wave movement is often used to express courage in facing change and the beauty of nature.

2. A Piece of Wire with Both Ends Tied

A string with both ends tied together, we vibrate and give it an initial transverse velocity $g(x) = \frac{x(x-l)}{8}$ and an initial displacement $f(x) = 0$. The particular solution of the wave equation in that case is

$$u(x,t) = \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi Ct}{l}\right) + b_n \sin\left(\frac{n\pi Ct}{l}\right)\right]\sin\left(\frac{n\pi x}{l}\right).$$

Next, by substituting the initial values $f(x)$ and $g(x)$, we obtain

$$a_n = \frac{2}{l}\int_0^l 0 \sin\left(\frac{n\pi x}{l}\right) dx = 0,$$

$$b_n = \frac{l^3}{2(n\pi)^4 C}(1-(-1)^n).$$

Thus obtained

$$u(x,t) = \frac{l^3}{2\pi^4 C}\sum_{n=1}^{\infty}\frac{(1-(-1)^n)}{n^4}\sin\left(\frac{n\pi Ct}{l}\right)\sin\left(\frac{n\pi x}{l}\right).$$

For an even value of n, the result is $(1-(-1)^n) = 0$, so the solution will also be 0. In order for $u(x,t)$ to not be 0, then n must be an odd number. We substitute $n = 2m-1$ with $m = 1,2,\ldots$. The solution is

$$u(x,t) = \frac{l^3}{2\pi^4 C}\sum_{m=1}^{\infty}\frac{(1-(-1)^{(2m-1)})}{(2m-1)^4}\sin\left(\frac{(2m-1)\pi Ct}{l}\right)\sin\left(\frac{(2m-1)\pi x}{l}\right). \quad (3)$$

When displayed in two dimensions, it will look like Figure 2. At points $x = 0$ and $x = 2$, there is no deviation because both points are the end points of the tied string. Points on the x-axis other than points $x = 0$ and $x = 2$, all experience changes in deviation as indicated by the color differences in Figure 2.

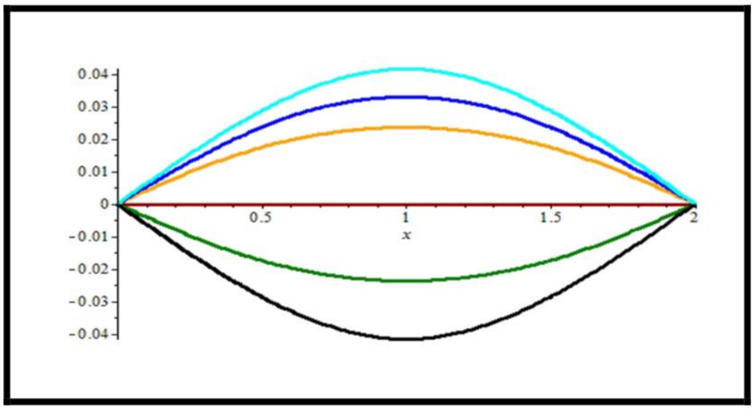

Figure 2. Wave Movement at Various Times $t = 0$ to $t = 1.5$ [1]

a. The Philosophical Meaning of a String Tied at Both Ends

The meaning of the designs in Figure 2 reflects the beauty and dynamics of the universe and shows that life is always changing, but still beautiful and harmonious. The design also reflects the symbolism of inevitable change, the courage to face challenges. In this motif, waves become a metaphor to convey a message about life that is always changing but still harmonious and beautiful.

3. Wave Deviation

A string with both ends tied together vibrates with an initial transverse velocity $g(x) = 0$ and an initial displacement $f(x)$ with

$$f(x) = \begin{cases} 0, & 0 \leq x \leq \frac{l}{4} \\ 4h\left(\frac{x}{l} - \frac{1}{4}\right), & \frac{l}{4} < x \leq \frac{l}{2} \\ 4h\left(\frac{3}{4} - \frac{x}{l}\right), & \frac{l}{2} < x \leq \frac{3l}{4} \\ 0, & \frac{3l}{4} < x \leq l \end{cases} \tag{4}$$

and h is the deflection height. The particular solution of the wave equation is

$$u(x,t) = \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{n\pi Ct}{l}\right) + b_n \sin\left(\frac{n\pi Ct}{l}\right)\right] \sin\left(\frac{n\pi x}{l}\right).$$

where

$$a_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

and

$$b_n = \frac{2}{n\pi C} \int_0^l g(x) \sin\left(\frac{n\pi x}{l}\right) dx.$$

Next, by substituting the initial values $f(x)$ and $g(x)$, we obtain

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$$a_n = \frac{32h}{(n\pi)^2} \sin\left(\frac{n\pi}{2}\right)\left[\sin^2\left(\frac{n\pi}{8}\right)\right]$$

$$b_n = \frac{2}{n\pi C}\int_0^l 0 \sin\left(\frac{n\pi x}{l}\right) dx = 0.$$

Thus obtained

$$u(x,t) = \frac{32h}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\left(\frac{n\pi}{2}\right)\sin^2\left(\frac{n\pi}{8}\right)\sin\left(\frac{n\pi x}{l}\right)\cos\left(\frac{n\pi Ct}{l}\right).$$

For even values of n, $\sin\left(\frac{n\pi}{2}\right)$ is 0, which results in $u(x,t) = 0$. So that $u(x,t)$ is not 0, n must be an odd number. Substitute $n = 2m - 1$ with $m = 1,2, \ldots$, so that

$$\sin\left(\frac{n\pi}{2}\right) = \sin\left(\frac{(2m-1)\pi}{2}\right) = (-1)^{m+1}.$$

Thus, the solution is

$$u(x,t) = \frac{32h}{\pi^2}\sum_{m=1}^{\infty}\frac{(-1)^{m+1}}{(2m-1)^2}\sin^2\left(\frac{(2m-1)\pi}{8}\right)\sin\left(\frac{(2m-1)\pi x}{l}\right)\cos\left(\frac{(2m-1)\pi Ct}{l}\right). \quad (5)$$

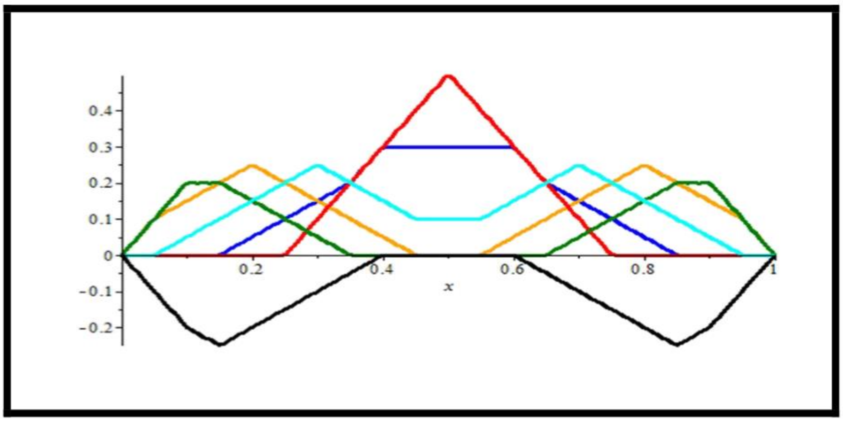

Figure 3. Wave Movement at Various Times $t = 0$ to $t = 0.65$ [1]

From Figure 3, we can see the difference in the form of deviation that occurs at each point x due to changes in time. When $t = 0$, the form of deviation that occurs is the initial deviation given as the initial value. For $0 < t \leq 0.65$, there is a change in the form of the initial deviation. The wave deviation that occurs has a maximum value of $u(x,t) = 0.5$. This shows that the maximum deviation (i.e. $h = 0.5$) does not exceed the specified initial deviation. At both end points, namely at points $x = 0$ and $x = 1$, no deviation occurs because the string is tied to both end points.

a. The Philosophical Meaning of Wave Deviation

The symbolism behind this motif depicts the concept of diversity, where differences in views or changes in relationships between individuals can be reflected. The design invites us to appreciate the complexity of change and to appreciate diversity with full awareness and to live differences in balance. Through this motif, it not only becomes a beautiful art medium, but

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also inspires us to live differences in balance and understand that true harmony lies in respecting diversity.

4. Wave Partition

Figure 4 shows the partition of a vibrating string. Let u denote the wave deviation so that $u(x)$ is the deviation at point x and $u(x+\Delta x)$ is the deviation $x+\Delta x$. We assume that the string particle moves only in the vertical direction, then the resultant force acting on the horizontal axis is zero, namely

$$\sum F_x = T(x+\Delta x)\cos\theta_2 - T(x)\cos\theta_1 = 0 \tag{7}$$

The tension on the string section is

$$T(x) = \frac{T}{\cos\theta_1}$$
$$T(x+\Delta x) = \frac{T}{\cos\theta_2}.$$

The resultant force in the vertical direction is

$$\sum F_u = T(x+\Delta x)\sin\theta_2 - T(x)\sin\theta_1 = T\tan\theta_2 - T\tan\theta_1$$

Since we are going to calculate the tangent when $\Delta t \to 0$, we express it in partial form $\frac{\partial u}{\partial x}$. Thus we obtain

$$\sum F_u = T\left(\frac{\partial u}{\partial x}(x+\Delta x,t) - \frac{\partial u}{\partial x}(x,t)\right).$$

Based on Newton's second law, we know

$$\sum F_u = \Delta m.a,$$

where $\Delta m = \mu.\Delta x$. Acceleration a is defined as the rate of change of velocity with respect to instantaneous time, so that

$$\sum F_u = \mu.\Delta x.\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial t}\right) = \mu.\Delta x.\frac{\partial^2 u}{\partial t^2}.$$

Thus, we obtain the relationship

$$\mu\frac{\partial^2 u}{\partial t^2} = T\frac{\left(\frac{\partial u}{\partial x}(x+\Delta x,t) - \frac{\partial u}{\partial x}(x,t)\right)}{\Delta x}$$

If $\Delta x \to 0$, then by the definition of partial derivatives, we obtain

$$\frac{\partial^2 u}{\partial t^2} = \frac{T}{\mu}\frac{\partial^2 u}{\partial x^2}$$

or we can write

$$\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}, \quad C = \sqrt{\frac{T}{\mu}}. \tag{8}$$

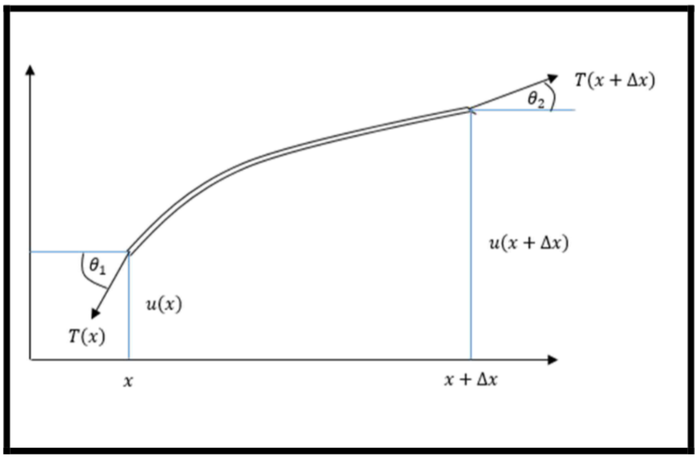

Figure 4. Partition of a Vibrating String [1]

a. Philosophical Meaning of Wave Partition

The batik motif depicting a partition of a piece of vibrating string (wire) has an interesting and profound meaning. Vibrating strings can represent changes in life, depicting that life is always in a state of flux. This motif reflects the concept of change, courage to face challenges, and beauty in diversity. Batik designs like this often invite us to realize and understand that in every journey of life, there is uniqueness and beauty that can be found, and inspire us to continue to adapt and grow in change.

5. Batik Process

The batik process is as follows.

a. The process of drawing motifs on Tracing Paper (Figure 5)
b. The process of drawing motifs on Primisima cloth (Figure 6)
c. Batik process. Figure 7 is a finished batik motif.

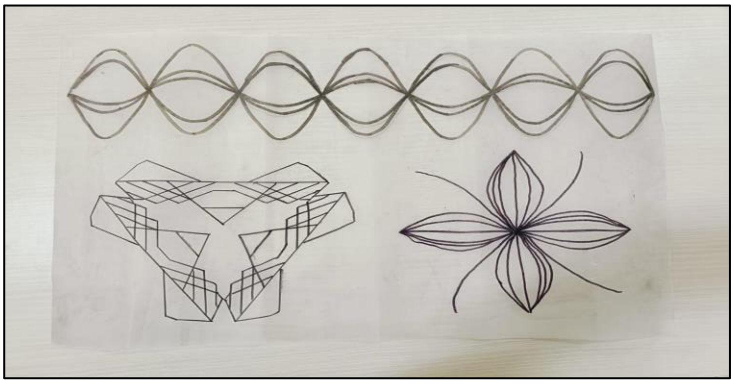

Figure 5. Batik Motif on Paper

Figure 6. Motifs on Primisima fabric

Figure 7. Waves in Batik Motifs

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Simpulan

The batik motifs taken from the movement of waves reflect a deep philosophy about life and change. Every aspect of the wave movement, both mathematically and symbolically, illustrates that even though change is inevitable, life can still maintain its beauty. Waves as a symbol of change and harmony remind us to respect diversity and celebrate the complexity in the process of change. Through designs inspired by waves, we are invited to understand and appreciate the uniqueness in every journey of life and adapt to change with a positive spirit.

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