International Journal on Emerging Mathematics Education (IJEME)
Vol. 7, No. 2, September 2023, pp. 109-118
P-ISSN: 2549-4996, E-ISSN: 2548-5806, DOI: http://dx.doi.org/10.12928/ijeme.v7i2.29298

Development of A Rubric for Assessing Multiplicative Thinking in Primary Schools in Zambia

Arthur Mungalu*
Graduate School for International Development and Cooperation, Hiroshima University
e-mail: mungalua@gmail.com

Abstract

The present study details the development of a rubric to assess Multiplicative Thinking in Zambia. Using data from a quasi-experiment involving 207 students in grades 3 to 5 from two schools, the study aimed at creating a rubric that accurately captures the range of students' responses to a whole number multiplication and division test. The test assessed 'reading patterns', 'drawing patterns', 'daily context', 'commutative property', and 'inverse relation'. The researcher examined students' external representations (illustrative and numeric) to understand and categorize their internal Multiplicative Thinking schemes using inductive content analysis. Findings revealed that binary assessment (correct or incorrect) was limiting; rather, including the 'partially correct' as well as 'no response' categories would capture the nuances of students' abilities. Teachers may adopt and adapt the rubric in their formative assessment in mathematics education. The study contributes to research in Multiplicative Thinking by adding the Zambian case to the ongoing discussion.

Keywords: assessment; multiplicative structure; multiplicative thinking; representation; rubric

How to Cite: Mungalu, A. (2023). Development of A Rubric for Assessing Multiplicative Thinking in Primary Schools in Zambia. *International Journal on Emerging Mathematics Education, 7*(2), 109-118. http://dx.doi.org/10.12928/ijeme.v7i2.29298

INTRODUCTION

Multiplicative Thinking

Multiplicative Thinking (hereafter, MT) is one of the 'big ideas' in mathematics education connecting various mathematical concepts such as proportional, algebraic, and functional reasoning (Hurst & Hurrell, 2014; Siegler et al., 2012; Siemon et al., 2006). Research shows that students struggle with these concepts due to insufficient development of an equal group structure in early concept formation – essentially, lacking MT (Mulligan & Mitchelmore, 1997). For instance, in Zambia, 6th grade students have minimal or no exposure to mathematical skills involving fractions, ratios, proportions, and algebra. Most students excel in addition and subtraction which define the emergent and basic numeracy levels in SACMEQ II, III, and IV (Morris, 2021). This issue stems from an over-reliance on counting-in-ones and additive strategies to solve problems that require composite relations between quantities (Baba et al., 2019; Nakawa, 2012; Uchida, 2009).

Multiplicative Thinking involves the ability to construct, coordinate, and represent composite units when dealing with contextual problems, the commutative property of multiplication, and the inverse relation between multiplication and division (Clark & Kamii, 1996; Hurst, 2017; Siemon et al., 2006). However, since traditional models – such as counting-in-ones, repeated addition, and subtraction – persist as "implicit and primitive" models for understanding whole number multiplication and division, respectively (Kaufmann, 2018), formative classroom assessment procedures

Received July 9, 2023; Revised August 3, 2023; Accepted September 17, 2023

must evolve to focus not just on correct or incorrect answers, but also on capturing students' underlying thought processes.

Classroom Assessment

In educational contexts, assessment plays a dual role. Firstly, it involves gathering information for system monitoring, program evaluation, and student placement – particularly relevant in large-scale assessments (Suurtamm et al., 2016). Secondly, assessment provides feedback to inform the modification of teaching and learning activities – a dimension known as classroom assessment (Chigonga, 2020; Nortvedt & Buchholtz, 2018). These two facets are commonly referred to as 'assessment of learning' (summative) and 'assessment for/as learning' (formative), respectively.

Contemporary perspectives on assessment transcend the measurement of learning outcomes emphasizing formative gathering of information to inform and improve teaching and learning processes (Baird et al., 2014; De Lange, 2007). Furthermore, involving students (and parents) in the assessment process is widely encouraged (Klenowski, 2009; National Council of Teachers of Mathematics [NCTM], 2014).

Aligned with this view of assessment, Zambia Education Policy documents have emphasized the importance of formative classroom assessment in enhancing learning outcomes (MoE, 1996; MoGE, 2013, 2017). Emphasis is placed on engaging students through self-assessment and peer assessment, enabling them to identify their abilities and opportunities for improvement. Further, the curriculum framework (MoGE, 2013, undergoing revision) underscored the need for teachers to share success criteria with students including ensuring fair and consistent evaluation of students' work through well-defined rubrics. Accordingly, the Zambia Education Curriculum Framework (MoE, 2023, undergoing validation) mandates the implementation of School-Based Assessment (SBA) for all learners in grades 4, 5, and 6 to be planned collaboratively by subject teachers and contribute towards the final grade of each student.

Despite this emphasis on formative assessment in policy documents, research reveals gaps in implementation. For example, Kapambwe (2010) examined the implementation of School-Based Continuous Assessment (SBCA) and reported that teachers faced challenges in embracing the shift from traditional objective-based assessment to outcomes-based assessment underpinned by SBCA. Also, Kakupa et al., (2019) collected questionnaire and interview data from teachers along with focus group discussions with students from three primary schools in Lusaka District focusing on the implementation of Formative Assessment (FA). The study revealed that learners in schools that implemented SBCA and FA achieved higher scores compared to their counterparts in schools without such programs. Factors such as class size, high pupil-teacher ratio, inadequate resources, and student absenteeism emerged as major impediments to the successful implementation of SBCA and FA. Other literature cites generic descriptions of formative assessment and insufficient teacher knowledge of rubric development and effective feedback provision impede the practical application of formative assessment (Hodgson et al., 2014; Siemon, Callingham & Day, 2021).

There is a dearth of literature on assessment in the Zambian context especially focussing on solutions (rather than pinpointing challenges) to enhance teachers' capacity to successfully implement SBCA as is envisaged by the policy documents. The literature reviewed highlighted the frequency of application of SBCA/FA without addressing "how assessment is done" whether using rubrics (as stressed by policy documents) or simply correct answer-only marking (Kakupa et al., 2019; Kapambwe, 2010). This study contributes to this discussion by proposing an evidence-based rubric development process for assessing Multiplicative Thinking in primary schools in Zambia. The study draws on experiences from a quasi-experiment study conducted among 207 3 rd to 5th graders from two schools in the Southern Province of Zambia. It addresses the question: What rubric can capture the variety of learners' responses to multiplication and division of whole numbers?.

Rather than being generic, the study brings to the fore subject and topic specificity in the realm of assessment for/as/of learning clarifying what students can do or cannot do in MT. This may provide opportunities for targeted teaching of whole number multiplication and division to enhance the shift from additive to MT, a current issue in mathematics education in Zambia.

RESEARCH METHOD

The study was exploratory allowing the researcher to investigate, gain broader insights, and identify patterns in students' responses to a MT test. The test drew on the conceptual framework (Hurst, 2017) and the Zambia Primary School Syllabus (MoGE, 2013) from the lens of mutual translations between representation modes (Mainali, 2021; Nakahara, 2008). The conceptual framework emphasizes Array Patterns as thinking tools to enhance learners' MT by connecting daily context, commutative property, and the inverse relation between multiplication and division. Rather than treating calculations, properties, and representations in isolation, the study integrated them to assess students' MT from the viewpoint of connections (Larsson, 2015).

The structure of the individual tasks for the test was adapted from relevant literature on MT (Almeida & Pietropaolo, 2022; Hurst, 2017; Götze & Baiker, 2021) to suit the Zambian context. For instance, a task inspired by Götze and Baiker (2021) read: "Construct multiplication array and calculate. Explain the pattern (a) 2 • 4 (b) 5 • 3". To address limitations in the familiar language/language of play (per the language policy – MoGE, 2013), we replaced "array" with "pattern" and adapted the multiplication notation to (×). Thus, our task read: "Use circles to draw a pattern that suits the mathematical expression 4×5 and write down the answer". The explanation aspect was reserved for the follow-up clinical interview (beyond this study's scope).

To ensure validity the test development drew on the literature on MT and underwent an iterative process involving two pilot sessions, peer and expert review to inform revisions leading to the final version (See Appendix 1). The Cronbach's Alpha showed that the test was reliable (α=.802). The test was translated into the familiar language/language of play, aligning with the language of instruction policy (MoGE, 2013, 2020). Table 1. shows the linkage between the assessed concepts, representation mode, and objective.

Table 1. Relation between concept, representation, and objective

Task No.	Concept	Representation	Objective
1.	Read pattern (×)	I→S	Write the multiplication number sentence that suits the given 5 by 3 array pattern.
2.	Draw pattern	S→I	Draw an array (pattern) that suits the expression 4×5.
3.	Daily context	L→I→S	Model the given multiplicative situation using an array (pattern) and find the answer.

4.	Commutativity (Identify commutated array)	I→L	Identify the rotated (commutative) array pattern.
5.	Commutativity (Identify commutated expression)	S→L	Identify the expression that illustrates the commutative property.
6.	Read drawn pattern (v?)	I→S	Write the division number sentence that suits the given 5 by 3 array pattern.
7.	Daily context (Partitive division)	L→I→S	Model the given multiplicative situation using an array pattern and find the answer.
8.	Daily context (Quotative division)	L→I→S	Model the given multiplicative situation using an array pattern and find the answer.
9.(a)	Draw pattern	L→I	(a) Draw an array that suits the given linguistic expression.
(b)	Read pattern	L/I→S	(b) Write the multiplication sentence that suits the given expression/array pattern.
(c)	Relate multiplication to division	S→S	(c) Write down the quotients for the given division sentences.

Note: I – Illustrative representation; L – Linguistic representation; S – Symbolic representation.

The finalized test was administered to 207 students in grades 3 to 5 aged 7 to 16 years (M=10.64, SD=1.54) from two schools in Southern Province, Zambia comprising a diverse socio-economic status of students. The province was chosen for its prevalence of counting and additive strategies. Participating schools were recommended by the District Education Office, while the school authorities identified the classes to be involved in the study based on their knowledge of class teachers' interest in innovative programs. The researcher ensured students understood each task by reading it aloud twice and allowing sufficient time for responses.

Students' external representations in the form of graphical drawings and numeric expressions were examined to understand their internal MT schemes. Responses were coded and categorized based on structural patterns related to the multiplicative structure – the relationships and patterns of organization of the elements in the representation of multiplication and division. The emerging categories were peer-reviewed and revised for reliability.

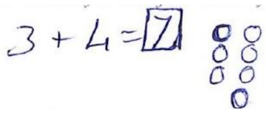
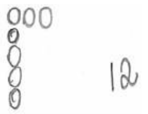
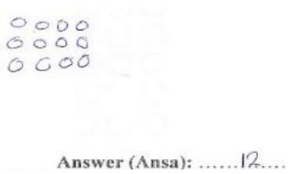
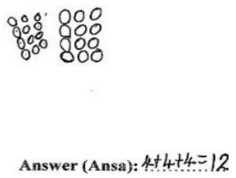
RESULTS AND DISCUSSION

In the content analysis of students' responses, blank answer spaces were labeled as 'no attempt'. Written responses were categorized as incorrect, partially correct, or correct.

Except for the partially correct category, the other categories resonated with those identified by Mulligan and Mitchelmore (2009) but, were inconsistent with the analysis by Hurst (2017) who focused on the proportions of correct responses. A major finding of this study is the identification of the 'partially correct' category. The following example of task 3 illustrates this category: "Munene planted 3 rows of cabbage plants. Each row had 4 plants. How many cabbage plants did he plant altogether? (Use circles to draw a pattern to illustrate your answer)".

Some students' illustrative representations included the relevant factors (three and four) but were incomplete (see Table 2). Götze and Baiker (2021) classified these as inappropriate graphical realizations. This study, however, identified two levels of numeric representations from the partial arrays: Incorrect answers from adding factors and correct answers from iterating factors (e.g., 3 four times or 4 three times). This distinction between learners' capabilities can inform targeted teaching and remediation to enhance students' MT.

Table 2. Typical examples of each category and descriptor

Category	Descriptor/Typical example	Score
No Response	Answer space is blank.	0
Incorrect	MS is not evident in the diagram with an incorrect answer. 	1
Partially Correct	MS is not evident in the diagram with a correct answer/MS is evident in the diagram with an incorrect answer. 	2
Correct	MS is evident in the diagram with a correct answer.  	3

Note: MS – Multiplicative Structure

Further analysis revealed that the categories of *incorrect*, *partially correct*, and *correct* align with the 'pre-structural stage (*PRS*)', 'emergent stage (ES)', & 'partial structural stage (PS)', and 'structural development stage (*S*)' respectively in stages of structural development in Mulligan & Mitchelmore (2009). Consequently, the 'partially correct' category exhibits traits of both emerging and partial Multiplicative Thinking.

Tasks were grouped according to structure: Read Pattern (RP), Draw Pattern (DP), Daily Context (DC), Commutative Property (CP), and Inverse Relation (IR) enabling the researcher to tailor descriptors for each task type in the rubric. For instance, the commutative property evaluated students' ability to recognize both the commutative pattern and expression. Consequently, the descriptors for the two aspects were merged into one that reflected both aspects. Based on such considerations, the rubric was summarised by refining the descriptors to reflect all aspects of the tasks in each group as shown in Table 3.

Table 3. Summary of rubric

Code & Task No.	**Category and Descriptor**			
	No attempt (0)	**Incorrect (1)**	**Partially correct (2)**	**Correct (3)**
RP 1, 6, 9(b)		MS of Pattern not evident in × or *v*? sentence/expression or uses an incorrect operation e.g., × for *v*? or writes total amount only	MS of the Pattern is evident in the × or *v*? expression/Correct expression with an incorrect dividend.	MS of the Pattern is evident in the multiplication or division sentence.
DP 2, 9(a)		MS is not evident in the Pattern (with incorrect/no answer). *Brackets () hold for task 2	MS is evident in the Pattern (with incorrect/ no answer)/MS is not evident in the Pattern with correct answer) *Brackets () hold for task 2	MS is evident in the Pattern (with a correct answer) *Brackets () hold for task 2
DC 3, 7, 8	Answer space is blank	MS is not evident in the Pattern with an incorrect answer.	MS is not evident in the Pattern (no diagram) with a correct answer/MS is evident in the diagram with an incorrect answer.	MS is evident in the Pattern with a correct answer.
CP 4, 5..		MS of circled Pattern/expression does not illustrate commutative of MS of given one.	MS of one circled pattern/expression illustrates commutative of MS of given one but the other does not (Circles correct and incorrect one)	MS of circled pattern/expression illustrates commutative of MS of given one.
IR		None of the two quotients is correct.	Only one quotient is correct	Both quotients are correct

Note: MS – multiplicative structure, Task No. – Task number

The following considerations, based on literature (Götze & Baiker, 2021; Hurst, 2017; Mulligan & Mitchelmore, 2009), were made during rubric development and may be useful for teachers adapting the rubric. The test items assessed multiple abilities through the translation between representation modes, making binary measurement (correct or incorrect) inadequate.

(i) If the numeric representation was incorrect or incomplete, the response was judged as 'partially correct', even if the illustrative representation was correct.
(ii) For illustrative representation, both equal groups and rectangular arrays were acceptable.
(iii) Commutative patterns or expressions were accepted for multiplication tasks.
(iv) Repeated addition sums (e.g., $5 + 5 + 5$ for a 3 by 5 array) were categorized as 'partially correct' along with the multiplication expression 3×5, as both coordinate the multiplier (3) and the multiplicand (5) but do not show the total amount (product).
(v) Either grouping by divisor or grouping by quotient was accepted for division tasks.

CONCLUSION

This paper outlined the creation of a rubric for assessing Multiplicative Thinking (MT) in Zambia. By analyzing students' responses to a test, it was established that assessing responses as either 'correct' or 'incorrect' was limiting. Therefore, including the 'partially correct' and 'no answer' categories would capture the nuances of students' MT abilities.

The rubric development process can be adapted by teachers to create assessments for different concepts in alignment with the rubric-based formative assessment policy. Specifically, the rubric discussed in this paper can be utilized to assess students' MT capabilities with whole numbers. Further, the rubric highlights opportunities for targeted teaching to enhance students' MT. Besides, the paper adds the Zambian case to the discussion on MT which could be a basis for further research.

The study recommends triangulating test data with interview data for in-depth analysis of students' levels of MT. Further, the small sample size hinders generalization, calling for large-scale studies to confirm the results, particularly with larger whole numbers.

ACKNOWLEDGEMENT

Author could thank anyone for their support to the research, such as funder, participants, etc.

DECLARATION

Author Contribution

All authors contribute in the research process, such as collecting the data, analyzing the data, and writing the manuscript. All authors approved the final manuscript.

Funding Statement

This research did not receive any funding.

Conflict of Interest

Both authors declare that they have no competing interests.

Ethics Declaration

We as authors acknowledge that this work has been written based on ethical research that conforms with the regulations of our institutions and that we have obtained the permission from the relevant institutes when collecting data. We support the International Journal on Emerging Mathematics Education (IJEME) in maintaining high standards of personal conduct, practicing honesty in all our professional practices and endeavors.

REFERENCES

Almeida, R., & Pietropaolo, R. C. (2022). Early-Career Mathematics Teachers' Knowledge in the Multiplicative Conceptual Field. *Acta Scientiae*, *24*(2), 179-211.

Baba, T., Nakawa, N., Nkhata, B., Mungalu, A., Mudenda, B., Kaabo, E., Kosaka, M., Kusaka, S., Mambwe, B., Nkharamo, C. J., & Watanabe, K. (2019). The development of a numeracy assessment instrument and its application to primary schools in Zambia. *Zambia Journal of Teacher Professional Growth (ZJTPG), 5*(2), 72–92.

Baird, J. A., Hopfenbeck, T. N., Newton, P., Stobart, G., & Steen-Utheim, A. T. (2014). Assessment and Learning: State of the field review assessment and learning (case number 13/4697). Oslo: Knowledge Center for Education.

Chigonga, B. (2020). Formative Assessment in Mathematics Education in the Twenty-First Century. In K. George Fomunyam (Ed.), *Theorizing STEM Education in the 21st Century*. https://doi.org/10.5772/intechopen.88996.

Clark, F. B., & Kamii, C. (1996). Identification of Multiplicative Thinking in Children in Grades 1-5. *Journal for Research in Mathematics Education*, *27*(1), 41–51. https://doi.org/10.2307/749196.

De Lange, J. (2007). Large-scale assessment and mathematics education. *Second handbook of research on mathematics teaching and learning, 2*, 1111-1144.

Götze, D., & Baiker, A. (2021). Language-responsive support for multiplicative thinking as unitizing: Results of an intervention study in the second grade. *ZDM – Mathematics Education*, *53*(2), 263–275. https://doi.org/10.1007/s11858-020-01206-1.

Hodgson, J., Coe, R., Brown, M., & Küchemann, D. (2014). Improving students' understanding of multiplicative reasoning and algebra: Did the ICCAMS intervention work? In S. Pope (Ed.), Proceedings of the 8th British Congress of Mathematics Education, (pp. 167-174). https://bsrlm.org.uk/wpcontent/uploads/2016/09/BCME8-Full.pdf

Hurst, C. (2017). Children Have the Capacity to Think Multiplicatively, as long as .... *European Journal of STEM Education*, *2*(3), 07. https://doi.org/10.20897/ejsteme/78169.

Hurst, C., & Hurrell, D. (2014). Developing the Big Ideas of Number. *International Journal of Educational Studies in Mathematics*, *1*, 1–18. https://doi.org/10.17278/ijesim.2014.02.001.

Kapambwe, W. M. (2010). The implementation of school based continuous assessment (CA) in Zambia. *Educational Research and Reviews, 5*(3), 99. https://doi.org/10.5897/ERR09.049.

Kaufmann, O. T. (2018). The problem of distinguishing multiplicative from additive reasoning in primary school classroom context. *European Journal of Science and Mathematics Education*, *6*(3), 100–112. https://doi.org/10.30935/scimath/9526.

Klenowski, V. (2009). Assessment for Learning revisited: An Asia-Pacific perspective. *Assessment in Education: Principles, Policy & Practice. 16*. https://doi.org/10.1080/09695940903319646.

Larsson, K. (2015). Connections for learning multiplication. *In International Symposium: Elementary Maths Teaching (SEMT'15)*, Prague, Czech Republic, 202-211.

Mainali, B. (2020). Representation in Teaching and Learning Mathematics. *International Journal of Education in Mathematics, Science and Technology*, *9*(1), 1–21. https://doi.org/10.46328/ijemst.1111.

Ministry of Education (MoE) (2023). *Zambia Education Curriculum Framework (Validation Edition)*. Zambia: Curriculum Development Centre.

Ministry of Education, Science, Vocational Training and Early Education (2013). *Zambia Education Curriculum Framework*. Zambia: Curriculum Development Centre.

Ministry of General Education (MoGE) (2017). *Learning Achievement at the Middle Primary School Level: Zambia Grade 5 National Assessment Survey Report of 2016*. MoGE.

Ministry of General Education (MoGE) (2020). *Standardized Terminologies and Concepts for Mathematics and Science in Zambian Languages: ECE and Grades 1 – 4*. Zambia: Curriculum Development Centre.

Morris, A. (2021). *The SACMEQ IV Project International: a study of the conditions of schooling and the quality of education*.

Mulligan, J. T., & Mitchelmore, M. C. (1997). Young Children's Intuitive Models of Multiplication and Division. *Journal for Research in Mathematics Education*, *28*(3), 309–330. https://doi.org/10.2307/749783.

Mulligan, J., & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal*, *21*(2), 33–49. https://doi.org/10.1007/BF03217544.

Nakahara, T. (2008). Cultivating mathematical thinking through representation. In Utilizing the representational system. Talk given at the APEC-Tsukuba International Conference (III), Tsukuba, Japan. Retrieved November (Vol. 17, p. 2019).

Nakawa, N. (2012). Classroom interaction in Grade 5 and 6 mathematics classrooms in two basic schools in Zambia. In T. Tso (Ed.), *Proceedings of the 36th conference of the international group for the psychology of mathematics education, 3*, 257–264.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.

Nortvedt, G. A., & Buchholtz, N. (2018). Assessment in mathematics education: Responding to issues regarding methodology, policy, and equity. *ZDM*, *50*(4), 555–570. https://doi.org/10.1007/s11858-018-0963-z.

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., ... & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological science, 23*(7), 691-697. https://doi.org/10.1177/0956797612440101.

Siemon, D., Breed, M., Izard, J. & Virgona, J. (2006). The derivation of a Learning and Assessment Framework for Multiplicative Thinking, In J. Novotna, H. Moraova, M. Kratka & 12th N. Stehlikova (Eds.), *Proceedings of the 30th conference of the International Group for the Psychology of Mathematics Education*, 5, pp.113-120.

Siemon, D., Callingham, R., & Day, L. (2021). The development and validation of two new assessment options for multiplicative thinking. *In Y. H. Leong, B. Kaur, B. H. Choy, J. B. W. Yeo, & L. F. Wong (Eds.), Excellence in Mathematics Education: Foundations* *and Pathways (Proceedings of the 43rd Annual Conference of the Mathematics Education Research Group of Australasia)* Pp. 369-378. Singapore: MERGA.

Suurtamm, C., Thompson, D. R., Kim, R. Y., Moreno, L. D., Sayac, N., Schukajlow, S., Silver, E., Ufer, S., & Vos, P. (2016). *Assessment in Mathematics Education*. Springer International Publishing.

Uchida, T. (2009). The diagnostic evaluation of pupils' performance in basic school in Zambia: Focusing on discrimination of performance and educational suggestiveness. *Journal of International Cooperation in Education, 12*(2), 1-12.