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Developing Number Theory Textbook for Pre-Service Mathematics Teacher of International Program

[1]Rully Charitas Indra Prahmana, [1]Puguh Wahyu Prasetyo, [1]Afit Istiandaru, [2]Uha Isnaini, [1]Burhanudin Arif Nurnugroho
[1]Universitas Ahmad Dahlan, Jl. Ahmad Yani, Tamanan, Banguntapan, Bantul 55191 DIY, Indonesia
[2]Universitas Gadjah Mada, Bulaksumur, Caturtunggal, Depok Sleman 55281 DIY, Indonesia
e-mail: puguh.prasetyo@pmat.uad.ac.id

Abstract

The long-term objective of this research is to produce valid, practical, and effective textbooks for courses in international program of mathematics education of Universitas Ahmad Dahlan. For the short-term, this research aims to produce valid and practical textbook for number theory course. The validity was based on the relevancy towards the students' expected competence, while the practicality was measured from the implementation of learning using the textbook. This research used design research with the type of development studies. It followed four steps, i.e. (1) preliminary research, (2) prototyping stage, (3) summative evaluation, and (4) systemic reflection and documentation. In the preliminary research, we analyzed the need of the textbook in the international program and found that the textbook of number theory course is very needed. In the prototyping stage, we wrote the textbook based on the need analysis. The prototype was then discussed with expert and revised in the stage of summative evaluation and applied to the lecture meetings. The result was the final product of the textbook used for the stage of systemic reflection and documentation when we wrote all the results according to the preset research framework. Finally, the developed number theory textbook is valid and practical.

Keywords: design research, international program of mathematics education, number theory course, pre-service mathematics teacher

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INTRODUCTION

The Mathematics Education study program is one of the study programs in the Teacher Training and Education Faculty (FKIP) Ahmad Dahlan University (UAD) Yogyakarta. The UAD mathematics education study program has a vision to be a reliable and Islamic study program in producing intellectual, independent, highly competitive human resources in the global era, adapting quickly to the development of science and technology and professionals in mathematics education. In 2020, the UAD mathematics education study program admits 65 new students. This is a challenge for the UAD mathematics education study program to always provide superior academic services for students so that they can produce quality graduates.

Many factors determine the quality of education, but teachers are still seen as the main determining factor, because teachers are in charge of learning, determine the direction of achieving learning goals, and manage student learning (Amir, 2013; Kunandar, 2007; Jalal, 2007). It is hoped that the UAD mathematics education program

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graduates can become competent teachers in both pedagogy, professional, social, Islamic personality, and technology.

One of the profiles of graduates of UAD's mathematics education study program is graduates who are competitive in the global era. Many universities, both public and private, have been entrusted by the Ministry of Research, Technology and Higher Education to organize a Mathematics Education study program. In Yogyakarta, there are 106 private universities with 510 study programs, of which no less than eight universities have opened mathematics education study programs (Kopertis Wilayah V, 2015: 8). The data shows that the competition for mathematics education study program graduates in the world of work is very high so that superior competencies must be pursued by graduates of UAD mathematics education study programs as graduates in order to be able to compete.

The current excellence criteria are not only based on the Grade Point Average (GPA), but also the accompanying skills that are important for students. The UAD mathematics education study program has an international class program that offers a variety of superior services, including: (1) improving English language skills, (2) increasing skills using technology-based learning media, and (3) increasing high-order thinking skills which are manifested in participation students at the mathematics olympiad (UAD Mathematics Education Study Program, 2013). Thus, the ability to speak English is one of the superior aspects of the UAD mathematics education study program.

The curriculum of the UAD mathematics education study program has accommodated efforts to improve English language skills by providing several courses offered by the Mathematics Education Study Program in the 2018/2019 even semester and the 2019/2020 academic year which consists of the following subjects, Matrix Algebra, Spatial Geometry, Differential Calculus, Introduction to Computer Science, Elementary Statistics, History and Philosophy of Mathematics Education and several other courses that will be offered in odd and even semesters of the 2019/2020 Academic Year for international classes in the Mathematics Education Study Program.

The basic assumption is that students have an important ability to speak English. In the current era of globalization, English has become the lingua franca in the arena of international relations. There are approximately 479 million native speakers of English and more than 700 million people who can speak English as a second language or a foreign language (Artini, 2013). However, the scope of these capabilities must be adjusted according to needs. The purpose of ESP is to make students able to speak English according to their target situation (Javid, 2013). Therefore, the study material for international classes must accommodate the needs of prospective teachers in (1) reviewing mathematics education literature, (2) compiling written products on mathematics education, (3) communicating scientific papers in mathematics education forums, and (4) teaching mathematics in English.

Based on observations and researchers' unstructured interviews with some of the lecturers of the above courses, the researcher obtained information that these courses did not have adequate learning tools and were in accordance with the needs. Instructors have made efforts by administering assignments and projects while learning resources can be obtained through online sources. The scope of the material also does not have a standard pattern according to the competency targets that must be achieved by students.

One of the important efforts to take is to develop valid and practical course learning tools for international classes. Valid and practical learning tools can be a means of teaching students on courses for international classes effectively.

English for Spesific Purposes

English for Specific Purposes (ESP) has been known since the 1960s and has developed into an important branch of English studies (Javid, 2013). This is triggered by globalization which allows knowledge to be shared not only in a narrow scope, but beyond national and language boundaries. In its development, ESP programs are made with due observance of the needs, namely the target areas of the ESP.

The development of scientific fields such as science and humanities is different from one area to another. The complexity of the world's problems makes the transfer of information, knowledge, and technology in search of collective problem solutions important. English is important because it is able to bridge the differences in the mother tongue of each source of information.

The specificity of ESP also lies in the depth of the material studied. Statisticians do not need to learn as many English idioms as artists and writers need, nor do artists need to learn how to pronounce mathematical symbols in English.

Berkaitan dengan bahan ajar ESP, para pakar terkadang menemukan dilemma yang disebabkan oleh keharusan menyiapkan bahan ajar yang harus sesuai dengan bidang ilmu pembelajar dalam waktu singkat (Gatehouse, 2001). Singkatnya waktu membuat bahan ajar yang disusun menjadi kurang mengena pada konteks yang dihadapi. Persiapan yang matang perlu dilakukan agar bahan ajar yang digunakan valid, praktis, dan nantinya efektif.

International Program Courses

Mathematics is the study of numbers, sets of points, and various abstract elements together with their relations and operations that apply to them (Adenegan, 2010). Mathematics leads humans to discover concepts. However, new discoveries will not occur if learning is not carried out effectively through the application of adequate learning, effective human resources, and adequate physical facilities. Abstract mathematical characteristics should be taught by demonstration and practice (Okigbo and Osuafor, 2008). This is related to the cognitive development of students who are not automatically able to abstraction mathematical objects.

Mathematical objects that are abstract in nature make the complexity of the material quite high. Thus, an ESP is needed that is able to facilitate learners communicating mathematical objects using English. Many mathematical terms are very different from the common terms in everyday life. For example, a ball as a geometric object may not be translated into English as ball, but sphere. Likewise, both are translated as circumference, but the circumference of a circle is translated into the circumference of a circle, not the perimeter of a circle.

Lesson Plan and Material

Mulyasa (2009: 5) suggests that in carrying out learning, teachers must prepare learning tools consisting of a syllabus, lesson plans (RPP), student worksheets (LKS), student books, and assessment instruments that include cognitive, affective, and psychomotor. The researcher adopted this opinion by concluding that the learning tools in a lecture should include: (1) syllabus, (2) lecture program units (SAP), (3) student worksheets (LKM), (4) teaching materials, and (5) learning outcomes test. The

syllabus is a matrix that is used as a reference for SAP development which includes subject identity, learning materials, learning activities, competency achievement indicators, assessments, time allocation, and learning resources. SAP is a plan that describes the procedures and organizing learning to achieve predetermined learning goals. Meanwhile, the LKM is a guide to a series of activities designed to make it easier for students to learn, and the learning outcome test is a measuring tool used to measure students' completeness in achieving certain competencies.

RESEARCH METHOD

This study uses the Design Research research method with the type of Development Studies. Research with the type of Development Studies is an activity to develop design principles for practical field purposes (van den Akker, et al, 2006; Prahmana, 2017). This research consists of several stages: first, preliminary research, activities to analyze contexts and problems to develop a conceptual framework foundation through literature review; second, the prototyping stage, the activity of designing design guidelines, optimizing prototypes through design design, formative evaluation and revision of research results; third, summative evaluation, the process of evaluating the effectiveness of the implementation and use of prototypes; fourth, sytematic reflection and documentation, the activity of writing down the entire study to support the analysis, then specifying the design principles and analyzing the results by connecting with the predetermined frame of mind.

RESULTS AND DISCUSSION

In this study, researchers conducted context and problem analysis to develop a conceptual framework foundation before designing a prototype of teaching materials. The context and problems analyzed include the curriculum, material analysis and analysis of student characteristics. The results of the analysis are then used as a reference for designing the prototype and optimizing it through the design of the prototype design, formative evaluation and revision of the evaluation results. So that a prototype 1 teaching material is obtained in the form of a textbook consisting of 7 chapters which have been adjusted to the curriculum used in the International Class, Mathematics Education Study Program, Ahmad Dahlan University.

The seven chapters include first, chapter 1, namely Basic Concept with Submaterial Number System, Mathematical Introduction and Binomial Theorem; second, chapter 2, namely Divisibility with the submissions of Divisibility Relation, Greatest Common Divisior (GCD) and Least Common Multiple (LCM); third, chapter 3, namely Integers Bases with Integer Bases submaterial; fourth, chapter 4, namely Integer Factorization with the submaterial Prime Number and Unique Factorzation; fifth, chapter 5, namely Congruence with the submissions of Concept and Basic Properties and Application of Congruence; sixth, chapter 6, namely Diophantine Equation with Linear Congruence, Linear Diophantine Equation and System of Linear Congruences; seventh, chapter 7, namely Fermat and Wilson Theorem with the submission of Fermat Theorem and Wilson Theorem. In the teaching materials that have been designed, each chapter is accompanied by an explanation of the theory, examples and steps to prove the theory as well as practice questions, see Figure 1.

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1.3 Binomial Theorem

1.3.1 Binomial Expansion

A binomial is an expression of the form $(a+b)^n$. In general, the expansion of these binomials for various values of n can be obtained by multiplying $(a+b)$ for n times. We have

$$(a+b)^n = \underbrace{(a+b)\times\cdots\times(a+b)}_{n \text{ times}}$$

We note that every term in the expansion of $(a+b)^n$ appears as $a^{n-i}b^i$, for some $0 \le i \le n$. The coefficient of $a^{n-i}b^i$ comes from the multiplication of $n-i$ times of a and i times of b. This equals to the number of choosing $n-i$ of a from n number of a or equivalently the number of choosing i of b from n number of b. Thus, the coefficient of $a^{n-i}b^i$ is

$$\binom{n}{n-i} = \binom{n}{i}$$

where

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}, 0 \le r \le n.$$

Furthermore, we have the following table in the next page.

Homework Chapter 1

1. Show that $1+2+3+\ldots+n = \binom{n+1}{2}$
2. Prove the following statement using mathematical induction for every positive integer n.
   a $1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{1}{3}n(4n^2-1)$.
   b $1^3+2^3+3^3+\cdots+n^3 = (1+2+3+\cdots+n)^2$.
3. Prove that
$$1+2+3+4+\cdots+n = \binom{n+1}{2}\ !$$
4. Prove that for any $n \ge 1$, $\binom{n}{k} = \binom{n}{k+1}$ if and only if n is an odd number and $k = \frac{1}{2}(n-1)$.
5. Prove that $n\binom{n-1}{k} = (k+1)\binom{n}{k+1}\ !$
6. Let k, r, n be natural numbers such that $0 \le k \le r \le n$. Prove that
$$\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$$
7. Prove that
$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots + n\binom{n}{n} = n2^{n-1}$$
8. Determine the result of the following sums
$$n\binom{n}{1} + n\binom{n}{3} + n\binom{n}{5} + \ldots + n\binom{n}{n}$$
where n is an even integer.

Example 1.4 Use mathematical induction to prove that for every positive integer n, $2^{2n}-1$ is divisible by 3 denoted by $3 \mid (2^{2n}-1)$.

Solution. For every positive integer n, we set

$$S(n) : 3 \mid (2^{2n}-1).$$

Step 1. We have $2^2-1 = 3$. Thus $S(1) : 3 \mid 2^{(2)-1}$ is true.

Step 2. (Inductive step) Suppose that

$$S(k) : 3 \mid (2^{2k}-1)$$

is true for every positive integer k. This means there exist an integer x such that $2^{2k}-1 = 3x$.

Step 3. We want to prove

$$S(k+1) : 3 \mid (2^{2(k+1)}-1).$$

We have

$$\begin{aligned} 2^{2(k+1)}-1 &= 2^{2k+2}-1 \\ &= 4\times 2^{2k}-1 \\ &= (3+1)2^{2k}-1 \\ &= 3\times 2^{2k}+(2^{2k}-1) \\ &= 3\times 2^{2k}+3x \\ &= 3\times(2^{2k}+x). \end{aligned}$$

Thus, the statement

$$S(k+1) : 3 \mid (2^{2(k+1)}-1)$$

is true. By the principle of mathematical induction, this implies for every positive integer n,

$$S(n) : 3 \mid (2^{2n}-1).$$

Figure 1. Textbook prototype

Furthermore, the researcher evaluates the effectiveness of the prototype and the effectiveness of using the prototype. Evaluation is carried out by asking for validation from the expert with the results of validation, namely first regarding the content, validation results are obtained that the material in each chapter is in accordance with the content of the number theory course, the questions are in accordance with the material contained in the module and there are no concepts wrong or wrong; secondly regarding constructs, validation results are obtained that the material in each chapter has been able to develop mathematical literacy skills and mathematical problem solving abilities, consisting of interrelated concepts, inviting further concept development and the questions are in accordance with the level of material provided; Third, regarding language, the results of the validation show that in each chapter the language with spelling in English is used, the content is presented in clear language and does not cause multiple interpretations and the questions given are clear and straightforward.

The validator also gave revised comments, among others, first in section 1.1, problem 1.1 of question number 4-7 is similar, the difficulty level of question number 7 is the lowest and it is still not possible to understand why question number 10 is split into two questions; second in section 1.2, there are not many exercises in problem 1.2, question no 2 is too small in rank and in section 1,2,3 it explains the "approach" as its application, this has not appeared in the problem. third in subab 2.1, can not see the questions that test critical thinking skills; fourth in section 2.2, in problem 2.2 questions number 1,4 and 5 are good for understanding the concept of division and should be added with similar questions. fourth in section 2.3, in problem 2.3 question number 2 lacks brackets and is inconsistent. The validation results and revised comments from the validator are then used as a basis for researchers to make improvements. As for some parts that were repaired, namely in problem 1.1 the questions were reordered into question number 7 then question number 5, question number 6 and the next sequence question number 4. Then, in problem 1.2 a practice question was added to ensure that students understand the usefulness of the binomial theorem and the problem. It is also added about the binomial theorea application. In sub-chapter 2.1, a question about checking the division of certain numbers is added which is rarely found in other books and in sub-chapter 2.2 questions that analyze concepts are added.

CONCLUSION

The research results have shown that the number theory book has been valid from the quality of the material and the quality of the media based on the results of the validation and revised comments from the experts.

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