Statistical MOSFET Parameter Extraction with Parameter Selection for Minimal Point Measurement

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Abstract
A method to statistically extract MOSFET model parameters from a minimal number of transistor I(V) characteristic curve measurements, taken during fabrication process monitoring. It includes a sensitivity analysis of the model, test/measurement point selection, and a parameter extraction experiment on the process data. The actual extraction is based on a linear error model, the sensitivity of the MOSFET model with respect to the parameters, and Newton-Raphson iterations. Simulated results showed good accuracy of parameter extraction and I(V) curve fit for parameter deviations of up 20% from nominal values, including for a process shift of 10% from nominal.

Keywords: MOSFET statistical parameter extraction; process monitoring; test point selection

1. Introduction
In order to achieve efficiency in the Integrated Circuit manufacturing process, the effects of the fluctuations and uncertainties in the fabrication process must be taken into account in decisions made throughout the entire process. Analysis and subsequent characterization of the statistical process involved is the basis for a large portion of the work to increase yield efficiency. This includes statistical design methods [1, 2, 3], as well as the monitoring and control of the fabrication process itself, e.g. tuning the process to optimize yield; and detecting shifts and changes in the process that might adversely affect production [4, 5]. There has been a substantial body of research dedicated to finding the most appropriate way to represent the statistical profile of the process in a way that can be used to facilitate subsequent prediction techniques. One aspect of this is the characterization of MOSFET devices, including MOSFET DC parameter extraction, which is the focus of this work.

There are 2 main approaches to the extraction of MOSFET DC parameters. The first is CAD related parameter extraction, which is mostly done by nonlinear optimization methods [6, 7]. This has the advantage of good I(V) curve fit and thus, is well suited for simulation applications but it has a myriad of problems when statistical applications are called for. These methods are time-consuming, require many measurement points and often result in unrealistic or unphysical values of parameters [7]. The second approach, i.e. direct or analytical extraction methods, was developed as an answer to statistical needs, but there are still problems of consistency, lack of robustness and accuracy.

Statistical parameter extraction, which uses measurements obtained during Wafer Electrical Test (WET) in process monitoring, is basically used to monitor the process via device
characterization, by deriving the distributions of the parameters and using them for predictive and diagnostic purposes. These methods are faster, less computationally demanding and require much fewer measurement points than their optimization counterparts, making them suitable for statistical and on-line extraction, but they usually give a poor fit to measured I(V) curves, which is a disadvantage when the parameters are to be used for circuit simulation [7]. Thus, the motivation is to develop a statistical parameter extraction strategy that results in accurate parameter extraction as well as a good curve fit. This would enable applications that utilize circuit simulation, such as circuit performance prediction.

An important factor in statistical parameter extraction is the known nature of the fabrication process under investigation, which makes it possible to derive certain assumptions about the characteristics of the devices measured. This is the underlying motivation for the choice of a linear error model approach to statistical parameter extraction here. It is based on the concept of a nominal or typical device, representing the stable process, and errors or deviations thereof that can be defined in the parameter or output responses of the device in question. This perspective lends itself well to the Taylor expansion, which in the first order is equivalent to the linear model [8] and the sensitivity model [9].

Although the variations in the manufacturing process are ideally small, the linear model alone cannot guarantee accuracy in extracting parameters, as deviations may well exceed the few percent allowed by this approximation. In our approach to statistical parameter extraction, Newton’s method is applied to achieve accuracy in extracting the DC parameters. Test point selection is also addressed; unlike many analytical methods where test points can be changed or added during the measurement process when dictated by calculation results, in our parameter extraction strategy, all test points are predetermined and the actual extraction process can be conducted off-line. This is advantageous in terms of speed of measurement.

2. Methodology for Statistical Parameter Extraction Steps

Figure 1 shows a block diagram of the methodology used for the extraction of parameters from MOSFET I(V) curves obtained from MOSFET test structures during Wafer Electrical Test (WET) monitoring. The I(V) curves and MOSFET parameters of the nominal/typical device, as well as their standard deviations, are assumed to be known from historical data of the process. There are three main steps, in the methodology: sensitivity analysis, test point selection and parameter extraction.

Using this methodology we determine whether the parameters of interest can be extracted based on the rank of the sensitivity matrix; if not, we have to reconsider our parameters and the sensitivity matrix. If they are extractable, we determine candidate subsets of parameters to be extracted during process control and we select test points for these various subsets. The parameter extraction experiment is then conducted on data that represent the spread of the process and we select the set of parameters that is to be extracted during process control from the results of the experiment. The selection is based on the extraction error, the cumulative I(V) curve error and the circuit performance error obtained from the experiment. The three steps are described in the following sections.

2.1 Step I: Sensitivity Analysis of Parameters

We would like to extract the smallest number of parameters possible from the parameters of interest in order to minimize the amount of measurements that need be taken during process monitoring. In this step we must first establish that all the parameters of interest can be extracted independently. Once this has been determined, a sensitivity analysis is done to select sets of candidate parameters for on-line extraction. Below we elaborate on these two procedures.

In order to determine that the parameters can be extracted independently, let us first review the linear error model used. Besides having the m-point nominal set of I(V) curves from the nominal device of the process, we have the "deviated" curves, i.e. the set of I(V) curves of a MOSFET device to be extracted. The value of a parameter $p_i$ for the set of nominal curves is $p_{nom_i}$, whereas for a set of deviated curves the parameter $p_i$ is:

$$p_i = p_{nom_i} + \Delta p_i$$  \hspace{1cm} (1)
The deviated curves are related to the nominal device curves using the sensitivity matrix:

\[ Y_{\text{dev}} = Y_{\text{nom}} + S\Delta p + Y_{\text{error}} \]  

(2)

where: \( Y_{\text{dev}} \) is an \( m \times 1 \) deviated curve vector; \( S \) is the \( m \times n \) sensitivity matrix of the nominal curve \( Y_{\text{nom}} \); \( \Delta p \) is the \( n \)-vector of parameter deviations from their nominal values; and, \( Y_{\text{error}} \) is the \( m \times 1 \) curve error vector caused by the nonlinearity of the actual curve with respect to the parameters \( p \). Thus, the linear error approximation we will use is:

\[ Y_{\text{dev}} = Y_{\text{nom}} + S\Delta p \]  

(3)

The sensitivity matrix \( S \) consists of \( n \) columns representing the \( n \) parameters and \( m \) rows representing the \( m \) measurement points, where \( n \leq m \). In order that the sensitivity matrix reflects the realistic variations in each parameter in the manufacturing process, the sensitivity of \( Y_{\text{nom}} \) to each parameter is multiplied by the standard deviation of that parameter. Consequently, to maintain the integrity of the equation, the elements of the parameter deviation vector, \( \Delta p \), must likewise be normalized to the individual standard deviations of the parameters.

Thus, in our notation from here onwards, the element in \( S \) at the position \((i,j)\) is the sensitivity of \( Y_{\text{nom}} \) at the \( i^{th} \) point, with respect to the parameter \( p_{j} \), multiplied by the standard deviation of \( p_{j} \), \( \sigma_{p_{j}} \). 

---

Figure 1. Block Diagram of the parameter extraction methodology
\[
S = \begin{bmatrix}
\frac{\delta Y_1}{\delta p_1} \sigma_{p_1} & \cdots & \frac{\delta Y_k}{\delta p_k} \sigma_{p_k} \\
\vdots & \ddots & \vdots \\
\frac{\delta Y_m}{\delta p_1} \sigma_{p_1} & \cdots & \frac{\delta Y_m}{\delta p_k} \sigma_{p_k}
\end{bmatrix};
\]
\[
\Delta p = \begin{bmatrix}
\delta p_1 \\
\vdots \\
\delta p_n
\end{bmatrix}
\]
(4)

From equation (3) we obtain:
\[
\Delta Y = S \Delta p
\]
(5)

where \(\Delta Y\) is the \(m\)-vector of measured curve deviations from the nominal curve, \(Y_{dev} - Y_{nom}\). This is the form of the linear error model that will be used.

In order to be able to extract all the parameters in the above equation, the sensitivity matrix \(S\) must be full-rank. Parameters associated with linearly dependent columns of \(S\) are said to belong to the same ambiguity group [10]; changes in these parameters cannot be distinguished at the \(I(V)\) curve level and they cannot be extracted independently. Applying the Singular Value Decomposition (SVD) on \(S\), we can determine the rank of \(S\) and, if \(S\) is singular, we can determine the parameters belonging to any ambiguity groups. With SVD, we obtain:
\[
S = uvv^T
\]
(6)

where \(u\), \(w\) and \(v^T\) are the resulting matrices of the SVD.

In order to be able to extract all \(n\) parameters, the matrix \(S\) must be of rank \(n\). This will be apparent through the \(w\) matrix, as the rank of \(S\) is the number of non-zero singular values. Thus, any diagonal elements of \(w\) that are equal to zero indicate a rank of less than \(n\). If the rank of \(S\) is less than \(n\), then there exist linearly dependent columns in \(S\). This means that some parameters cannot be individually determined; they can only be determined to a dependency on each other. These parameters can be identified by examining the nullspace of \(S\); using SVD, the columns of the matrix \(v\) that are associated with any zero singular values of \(w\) form an orthogonal basis for the nullspace. These column vectors are used to form the matrix \(N\) such that:
\[
SN = 0
\]
(7)

A non-zero element in row \(i\) of \(N\) indicates that the parameter associated with column \(i\) in \(S\) belongs to an ambiguity group. Two parameters \(i\) and \(j\) belong to the same ambiguity group if the rows \(i\) and \(j\) in \(N\) contain non-zero elements and are not orthogonal to each other.

If the sensitivity matrix \(S\) is determined to be singular with rank \(r\), where \(r < n\), then in order to extract the parameters independently, either the value of \((n-r)\) parameters, representing all the ambiguity groups, would have to be known a priori; or new test points with different sensitivity equations, that would eliminate the linear dependence of the columns, would have to be introduced into the \(S\) matrix.

On occasion, the rank of \(S\) is not immediately obvious from the singular values found in \(w\). When the ratio of the largest to the smallest singular value, i.e. \(w_{\text{max}}/w_{\text{min}}\) is very large, the matrix is ill-conditioned, suggesting that the columns of \(S\) are not really independent. In this case, we can define a tolerance \(\eta\) such that if \(w_{\text{max}}/w_{\text{min}} > \eta\), \(w_{\text{min}} = 0\) and \(\text{rank}(S) = i-1\).

Once the non-singularity of the sensitivity matrix \(S\) has been established, we can conduct the sensitivity analysis to select which of the parameters are to be extracted. The sensitivity analysis will show the order of the parameters, from the one having the largest effect to the one with the smallest effect on the MOSFET \(I(V)\) curve characteristic.

The sensitivity analysis first finds the change in the curve characteristic, i.e. the MOSFET drain current \(I_{\text{drain}}\), with respect to the change in each parameter. In order to reflect realistic variations in the process, we define the process sensitivity \(S_p\) for each parameter as the change in output, \(I_{\text{drain}}\), when the parameter changes from one standard deviation below its nominal value to one standard deviation above its nominal value.
\[
S_{p_i} = I_{\text{drain}}(p_{nom} + \sigma_{p_i}) - I_{\text{drain}}(p_{nom} - \sigma_{p_i})
\]
(8)

where \(I_{\text{drain}}\) is the measured drain current; \(p_{nom}\) is the \(i\)th parameter at its nominal value; and, \(\sigma_{p_i}\) is the standard deviation of the parameter \(p\).

Thus, we see that each parameter’s \(S_p\) sensitivity reflects a change for that parameter in proportion to its standard deviation. If the standard deviation of a parameter is relatively large,
its sensitivity will reflect this and be larger accordingly. From here we can get a good idea of the most significant to the least significant parameter, in terms of its influence on the $I(V)$ characteristic $I_{diss}$ and hence, a tentative selection of the parameters to be extracted can be attempted.

2.2. Step II: Test Point Selection

In this step we select the points on the MOSFET $I(V)$ curves that are to be measured, henceforth called the test points. The test point selection is based on the nominal set of curves. Once the test points are selected, they are fixed as measurement points for subsequent measurements done on sets of "deviated" curves from devices for which we wish to extract parameters. Test point selection involves choosing a subset of $n$ test points from the $m$ candidate points of $Y_{nom}$. This can be done simply and effectively using QR factorization.

The test point selection is be done by applying QR factorization to the transpose of the sensitivity matrix $S^T$ [11]. The columns of $S$ represent the different candidate test points whereas the rows are associated with the individual parameters. QR factorization with pivoting is applied to $S^T$; this results in an $R$ matrix with decreasing absolute values of its diagonal elements and a permutation vector $E$ which gives the reordering of the $S^T$ columns needed to result in the above $R$ matrix. The first $n$ test points indicated in $E$ represent the selected test points.

2.3. Step III: Parameter Extraction and Feasibility Assessment

After selecting the test points to be used, the extraction procedure to be used is assessed on data representing the statistical process spread. From the results of the extraction, conclusions can be taken regarding the feasibility of the extraction method, including the best subset of parameters to be extracted during on-line process monitoring and control. The extraction procedure is described below.

Using the $n$ selected test points or rows of $S$, we obtain an $n \times n$ reduced sensitivity matrix $S_R$ and reduced $n$-vectors of curve deviations $\Delta Y_R$ and curve errors $Y_{error_R}$. Equation (2) becomes:

$$\Delta Y_R = S_R \Delta p + Y_{error_R}$$ (9)

Parameter extraction is done from Equation (5):

$$\Delta p_{extr} = S_R^{-1} \Delta Y_R$$ (10)

where $S_R^{-1}$ is the inverse of the non-singular reduced sensitivity matrix $S_R$ found in the previous step. Using Equation (9), the true value of $\Delta p$ is:

$$\Delta p_{true} = S_R^{-1} (\Delta Y_R + Y_{error_R})$$ (11)

So the parameter extraction error can be found from Equation (10) and (11):

$$p_{error} = \Delta p_{extr} - \Delta p_{true}$$

$$= S_R^{-1} Y_{error_R}$$ (12)

Thus, we see that the extraction error $p_{error}$ is determined by the inverse of the reduced sensitivity matrix and so, in general, the parameter with the smallest sensitivity will get the greatest portion of the curve error $Y_{error_R}$.

For small deviations in parameters ($\leq 3\%$), the extraction errors are usually tolerable, but when the deviations become big, the linear approximation in Equation (3) breaks down and the extraction errors become very large. In order to accommodate curves, and their corresponding parameters that deviate greatly from nominal values, Newton-Raphson iterations[12] are used in the extraction process. This is done as follows:

For iteration $i$:

$$p^{(i)} = p^{(i-1)} + \Delta p^{(i)}_{extr}$$ (13)

$\Delta p^{(i)}_{extr}$ can be substituted using equation (10), to obtain:

$$p^{(i)} = p^{(i-1)} + [S_R(p^{(i-1)})]^{-1} (\Delta Y_R^{(i-1)})$$
Note that we use the subscript $R$ to imply a reduced matrix or vector that includes only the selected test points. Initially, at iteration $i = 1$:

$$p^{(1)} = p^{(0)} + [S_R(p)]^{-1} \left( Y_{devR} - Y_R(p^{(0)}) \right)$$  

(15)

where $p^{(i)}$ is the vector of parameters extracted in this iteration, $p^{(0)}$ is the vector of nominal parameters $p_{nom}$, $S_R(p)$ is the sensitivity matrix for $p_{nom}$, $Y_{devR}$ the measured curve and $Y_R(p)$ is the nominal curve $Y_{nomR}$.

After the first extraction of Equation (15), a new reference curve $Y_R(p^{(1)})$ is generated using the extracted parameters $p^{(1)}$ and a new $\Delta Y_R^{(1)}$ vector is determined as the difference between the measured curve $Y_{devR}$ and this reference curve. New parameter values are then extracted and this is repeated iteratively until the extracted parameters converge.

From the results of the parameter extraction on the process data, the final determination of the parameters to be extracted on-line is made, and the feasibility of the extraction method is assessed based on the extraction errors and simulation of $I(V)$ curves using the extracted parameters.

3. Experimental Results

We applied the minimal point parameter extraction methodology to MOSFET $I(V)$ characteristic curves, which were simulated using the level 3 SPICE model, with 7 parameters deviating from their nominal values. The 7 parameters were $U_0$, $V_{TO}$, $V_{MAX}$, $THETA$, $GAMMA$, $KAPPA$ and $ETA$ and each set of $I(V)$ curves consisted of 130 simulation points. Each parameter was assumed to have a standard deviation in the process of 10% from its nominal value.

3.1. Step I: Sensitivity Analysis of Parameters

Initially, a sensitivity analysis was done to see the effects of the changes in the individual parameters on the nominal set of curves. We first determined whether all the parameters of interest could be extracted independently by examining the sensitivity matrix of the 7 parameters using SVD to diagnose its rank; in this case the sensitivity matrix was of full-rank. We then calculated the parametric sensitivity $Sp$ of the drain current, $I_{drain}$, with respect to each of the parameters, as can be seen in Table 1 in order from the largest to the smallest sensitivity.

<table>
<thead>
<tr>
<th>Table 1. Normalized parametric sensitivity $Sp$ of $I_{drain}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_0$</td>
</tr>
<tr>
<td>$Sp_{(I_{drain} \text{ sensitivity})}$</td>
</tr>
</tbody>
</table>

The parametric sensitivity $Sp$ is the cumulative change in output, i.e. $I_{drain}$, when the parameter $p_i$ changes from one standard deviation below its nominal value to one standard deviation above its nominal value. The cumulative change is the sum of the change in all 130 points of the curves. In this table the magnitude of the sensitivities have been normalized with respect to the largest sensitivity, i.e. the sensitivity relating to the parameter $U_0$.

Using this table we propose some candidate sets of parameters to be selected, as seen in Table 2. In general, a parameter with a greater $Sp$ sensitivity accounts for a greater portion of the curve deviation $\Delta Y$. This is because $Sp$ reflects the change in curve $\Delta Y$ that corresponds to a parameter's standard deviation in the manufacturing process. Accordingly, the parameters are chosen in order of their sensitivities, from largest to smallest. For faster but less accurate extractions fewer parameters are chosen; parameters with approximately the same magnitude of sensitivity are selected together. The set of parameters ultimately chosen depends on the circuits we are interested in simulating, the worst case extraction errors, as well as the number of parameters we can afford to extract in terms of the time spent on measurement and extraction.
Table 2. Candidate sets of selected parameters

<table>
<thead>
<tr>
<th>Number of parameters</th>
<th>Selected Parameters</th>
<th>Reasoning</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>UO</td>
<td>fastest, least accurate</td>
</tr>
<tr>
<td>3</td>
<td>UO, VTO, VMAX</td>
<td>fast, more accurate</td>
</tr>
<tr>
<td>5</td>
<td>UO, VTO, VMAX, THETA, GAMMA</td>
<td>slow, still more accurate</td>
</tr>
<tr>
<td>7</td>
<td>UO, VTO, VMAX, THETA, GAMMA, KAPPA, ETA</td>
<td>slowest, most accurate</td>
</tr>
</tbody>
</table>

3.2. Step II: Test Point Selection

In this step we select the test points from the 130 candidate points of the nominal set of MOSFET $I(V)$ curves. The test points selected will later be used as measurement points on the MOSFET test structures during process control. We select one point for each parameter to be extracted in order to keep the measurements to a minimum.

From the previous step we have a number of candidate sets of parameters to be selected, as seen in Table 2. We select the test points for each set using Stenbakken & Souders' method with QR Factorization, as described in Section 2.2. The selection results with QR factorization for the different parameter sets can be seen in Figure 2.

![Figure 2. Test points selected on the nominal curve with QR Factorization](image_url)

### a. Test Points for 7 and 3 Parameters with QR Factorization

### b. Test Points for 1 and 5 Parameters with QR Factorization

3.3. Step III: Parameter Extraction and Feasibility Assessment

A parameter extraction experiment was run to assess the application of the extraction method on data representing the statistical process spread. The samples of parameter combinations for each run of the extraction experiment were generated using the Latin Hypercube sampling method, for a total of 2000 samples. Each parameter was assumed to have a uniform probability distribution, with deviations between -20% to +20% of its nominal value. A run consists of the generation of the deviated curve, corresponding to the parameter combination of the run; and, the extraction procedure, which includes the generation of additional curves needed for the iterations required.

After measurement of the deviated curve according to the selected test points, the parameters of interest were extracted using Newton-Raphson iterations as described in section 2.3. For instances where only a subset $k$ of the 7 parameters were extracted, the extraction process was the same, with the $(7-k)$ parameters that were not extracted held at their nominal values in any extraction calculations. The results are shown in Table 3.

The extracted parameters obtained were used to re-simulate the deviated curves and the cumulative curve error was calculated. This error is also shown in the tables in order to give an idea of the curve error the extracted parameters will give when used for $I(V)$ curve simulation.

As is to be expected, extraction of all 7 parameters yields the lowest extraction errors for the parameter values as well as the lowest cumulative curve error. It also takes the most time, with the most test points measured and the most Newton Raphson iterations used to converge. Comparing the results of Table 3 to the candidate sets of parameters in Table 2, we see that, for an increase in speed, the subset containing 3 parameters (UO, VTO and VMAX) is the obvious choice as it has a small number of parameters to be extracted and, hence, test
points to be measured, as well as a small number of iterations needed for the extraction procedure, while still giving reasonable parameter extraction errors and a good cumulative curve error. The curve fit is also very good, as can be seen in Figure 3. The difference between the original $I(V)$ curve set and the set simulated using extracted parameters for the worst case is barely discernible.

Table 3. Mean Extraction Error

<table>
<thead>
<tr>
<th>No. of test points</th>
<th>Maximum iterations</th>
<th>Mean of extraction errors (%) of parameters extracted</th>
<th>Mean cumulative curve err (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$U_0$</td>
<td>$V_{T0}$</td>
<td>$V_{MAX}$</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>6.956</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2.750</td>
<td>18.91</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>2.214</td>
<td>2.523</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1.941</td>
<td>2.99</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.731</td>
<td>0.137</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>&lt;10^{-3}</td>
<td>&lt;10^{-3}</td>
</tr>
</tbody>
</table>

Assuming the choice of extracting 3 parameters during process monitoring, the results of the experiment using this extraction procedure is further investigated. The spread of the extraction errors for each parameter, $U_0$, $V_{MAX}$ and $V_{T0}$, can be seen in the respective histograms of Figure 4. For all three parameters the bulk of the extraction errors fall within the 5% range, though both $V_{MAX}$ and $V_{T0}$ show errors of up to about 10%. In Figure 4 we can also see that the extraction error is reasonably random with regard to the value of the parameter involved.

Next we investigate the ability of the 3 parameter extraction procedure to detect a shift in the process from nominal by 10%. The parameter extraction experiment is rerun; this time the samples representing the statistical process spread are generated with respect to the shifted process, i.e. nominal values that are 10% larger than the original nominal values. The original 3 parameter extraction procedure is then applied to the new data. Using Table 4 we can compare the means and standard deviations of the parameters extracted from the nominal and the shifted process. The errors of the extracted means and standard deviations of the parameters are small and thus, we can conclude that shifts in the process within this 10% range can be monitored via these statistics.

Table 4. Extraction of 3 parameters in the nominal and shifted process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$U_0$ mean</th>
<th>$U_0$ std-dev</th>
<th>$V_{T0}$ mean</th>
<th>$V_{T0}$ std-dev</th>
<th>$V_{MAX}$ mean</th>
<th>$V_{MAX}$ std-dev</th>
<th>$\Theta$ mean</th>
<th>$\Theta$ std-dev</th>
<th>$\Gamma$ mean</th>
<th>$\Gamma$ std-dev</th>
<th>$\Kappa$ mean</th>
<th>$\Kappa$ std-dev</th>
<th>$\eta$ mean</th>
<th>$\eta$ std-dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>0.06</td>
<td>0.0009</td>
<td>0.7668</td>
<td>0.0885</td>
<td>1.319e+5</td>
<td>1.524e+4</td>
<td>0.066</td>
<td>0.0078</td>
<td>0.843</td>
<td>0.097</td>
<td>1.452e+5</td>
<td>1.676e+4</td>
<td>0.07</td>
<td>2.30</td>
</tr>
<tr>
<td>extracted</td>
<td>0.06</td>
<td>0.0071</td>
<td>0.7667</td>
<td>0.0911</td>
<td>1.321e+5</td>
<td>1.597e+4</td>
<td>0.065</td>
<td>0.0076</td>
<td>0.840</td>
<td>0.100</td>
<td>1.412e+5</td>
<td>1.680e+4</td>
<td>0.02</td>
<td>2.91</td>
</tr>
<tr>
<td>error (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 5. Curve Error in the Nominal and Shifted Process

<table>
<thead>
<tr>
<th>Cumulative curve error (%)</th>
<th>mean</th>
<th>standard-deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>nominal process</td>
<td>0.2116</td>
<td>0.0597</td>
</tr>
<tr>
<td>shifted process</td>
<td>0.2681</td>
<td>1.4478</td>
</tr>
</tbody>
</table>
Statistical MOSFET Parameter Extraction with Parameter Selection for … (Marga Alisjahbana)

Figure 3. Original curve and curve simulated using extracted parameter

Table 5 gives the cumulative curve error of the $I(V)$ curves generated using the extracted parameters. These are small and differences between the true curves and those generated using extracted parameters are barely discernible.

4. Conclusion
The statistical parameter extraction methodology presented here is intended for use in circuit performance prediction, as shown in the example. The parameters obtained are better suited for use in circuit simulation than transistor characterization. This is because errors can be mathematically induced, in which case changes in the extracted parameters do not necessarily reflect physical phenomenon, compared to the direct extraction methods used for process
diagnosis. This is especially true when only a subset of the parameters are extracted; on the other hand, the $I(V)$ curves resulting from CAD simulation of the extracted parameters show a very good fit with a mean cumulative curve error of less than 0.2116% for 3 parameters extracted. Error in estimating the individual parameters was larger, with a mean parameter error of 2.214% for $U_0$, 2.523% for $V_{T0}$ and 2.76% for $V_{MAX}$ for the subset of 3 parameters extracted.

Results for the extraction of a selected subset of the full set of parameters were good and gave a very good $I(V)$ curve fit and thus, measurements could be held to a minimum during fabrication, subject to the measurement error encountered. Large measurement error would necessitate the addition of measurement points beyond the number of parameters extracted.

The extraction ability in a process with a shift of up to 10% from nominal was also good, with mean extraction errors of 1.7280% for $U_0$, 0.3829% for $V_{T0}$ and 2.6987% for $V_{MAX}$ for a subset of 3 parameters extracted and a mean cumulative curve error of 0.2681% for the simulated $I(V)$ curves.

References