Abstract

Metaheuristic algorithms are proven to be more effective on finding global optimum in numerous problems including the constrained optimization area. The algorithms have the capacity to prevail over many deficiencies in conventional algorithms. Besides of good quality of performance, some metaheuristic algorithms have limitations that may deteriorate by certain degree of difficulties especially in real-world application. Most of the real-world problems consist of constrained problem that is significantly important in modern engineering design and must be considered in order to perform any optimization task. Therefore, it is essential to compare the performance of the algorithm in diverse level of difficulties in constrained region. This paper introduces Tree Physiology Optimization (TPO) algorithm for solving constrained optimization problem and compares the performance with other existing metaheuristic algorithms. The constrained problems that are included in the comparison are three engineering design and nonlinear mathematical problems. The difficulties of each proposed problem are the function complexity, number of constraints, and dimension of variables. The performance measure of each algorithm is the statistical results of finding the global optimum and the convergence towards global optimum.

Keywords: metaheuristic algorithm, constrained problem, engineering design, tree physiology optimization, global optimum
compares the performance of TPO with other metaheuristic algorithms with constrained optimization problem. An introduction of TPO as metaheuristic algorithm for constrained optimization problem is introduced in Section 2. In Section 3, a short overview of other metaheuristic algorithms and the proposed constrained problem is presented. Section 4 compares the efficiency of each metaheuristic with the proposed problem and the last section summarizes a conclusion for this paper.

2. Tree Physiology Optimization

The Tree Physiology Optimization (TPO) algorithm is enthused from plant growth system [6]. The idea of TPO consists of two main components, which are: shoots- and roots growth. The shoots growth of any plant is driven by the light intensity as positive phototropism behaviour [7]. The plant shoots extend towards light in order to convert water and carbon dioxide (CO2) into carbon. Carbon is an essential source for plant especially for root growth. The propagation of the shoots is depending on the nutrients supplied by the roots system. Contrary, the roots counterparts consume carbon gained by the shoots system and grow towards soils as positive gravitropism behaviour in order to search for nutrients [8]. The shoot-root relationship is simplified as a Thornley-model [8-9]. Based on the model, shoots consumed nutrients and extend towards light for photosynthesis process and produces carbon, whereas roots consumed carbon gained from shoots system and elongate towards soil for nutrient absorption. This idea inspired an optimization process, which is referred as Tree Physiology Optimization [6]. The TPO algorithm is established with four equations that represents shoots extension, carbon gain, root elongation, and nutrient absorption. The shoots extension is defined as:

$$S_i^{kj} = S_i^{kj} + (S_{gbest} - S_i^{kj}) + \beta N_i^{kj}$$  \hspace{1cm} (1)

With $S_i^{kj}$ is the value of current shoot during ith iteration, of kth leafs and jth branches, $S_{gbest}$ is equivalent to global best value from all branches, $N_i^{kj}$ is the value of nutrient initiated by root, and $\beta$ is a diversification constant. Higher $\beta$ lead to more diversified shoots. Too big $\beta$ leads to dispersed shoots and may take longer time to converge, whereas too small $\beta$ might lead to less scattered of shoots allocation and thus resulted in local optimum. Each shoot undergo photosynthesis and converts into carbon gain. The value of carbon-gain corresponds to the deviation of individual shoot with its branch best as:

$$C_i^{kj} = \theta(S_{popbest} - S_i^{kj})$$  \hspace{1cm} (2)

$C_i^{kj}$ is current carbon gain, $S_{popbest}$ is best shoot of current branch. The value $\theta$ is equivalent to a power-law so as to reduce the randomness as iteration increases in a pattern of a monotonic decreasing function. Typical value for better convergence is 0.9. $C_i$ amplifies the root elongation in search for more nutrients as in (3). Therefore a good $\theta$ value lead to good amplification of roots distribution.

$$r_i^{kj} = r_i^{kj} + \alpha \cdot C_i^{kj} \cdot \epsilon$$  \hspace{1cm} (3)

$r_i^{kj}$ is equivalent to current root, $\alpha$ is an absorption parameter, $\epsilon$ is a random numbers. The root elongates into soil with a random motion. The value of $\alpha$ has the same objective as $\beta$ in shoot extension: to ensure a better diversification and convergence. The nutrient uptake is assumed as a factor of root elongation.

$$N_i^{kj} = \theta(r_i^{kj} - r_i^{kj})$$  \hspace{1cm} (4)

With $r_i^{kj}$ and $r_i^{kj}$ as the current and previous value of root respectively. Some effort in using TPO as optimization tool are applied in numerous application such as nonlinear ANFIS modeling [6].
neural network training [10], and PID tuning [11]. In nonlinear optimization problems, TPO outperformed other metaheuristic algorithm with lesser computation time [12].

3. Research Method

The performance of TPO is compared with other three metaheuristic algorithms in the constrained optimization problem. The overviews of other three metaheuristics are summarized in Table 1. The capability of proposed algorithms is verified with constrained optimization problem as shown in Table 2. These problems have different difficulties such as number of variables, number of constraints and dimension complexity.

Table 1. Overview of Four Metaheuristic Algorithms

<table>
<thead>
<tr>
<th>Num</th>
<th>Algorithm (year)</th>
<th>Main features</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Particle Swarm Optimization, PSO (1995)</td>
<td>Swarm-based : inspired by swarm movement of creatures. Two type of solution attraction: local and global best value per iteration with two equations, speed and position: $v_{i+1}^t = v_i^t + \alpha (g_i - x_i^t) + \beta_1 (x_i^* - x_i^t)$, $x_{i+1}^t = x_i^t + v_{i+1}^t$ [13]</td>
</tr>
<tr>
<td>2</td>
<td>Firefly Algorithm, FA (2007)</td>
<td>Inspired from fireflies flashing mechanism. Solution equivalent to attractiveness of flashing behaviour (fitness). Attraction of firefly i to firefly j with: $d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$</td>
</tr>
<tr>
<td>3</td>
<td>Cuckoo Search, CS (2009)</td>
<td>Inspired from brood parasitism behaviour of cuckoo bird. New solution is generated via levy flight: $x_{j+1}^i = x_j^i + \alpha \cdot L$ with L as levy flight. The fitness is correlated to randomly chosen best nest. A fraction of worst nest is eliminated by a probability factor [15]</td>
</tr>
</tbody>
</table>

Table 2 indicates the most widely used constrained optimization problems with three engineering design problems and one nonlinear mathematical problem.
The three engineering design problems for this benchmark are shown in Figure 1. The difficulties of each benchmark problem as tabulated in Table 2 are the function complexity, number of constraints, and dimension of variables. Each algorithm is set according to their nature of coding and searching mechanism as depicted in Table 3.

### Table 3. Algorithm Setting

<table>
<thead>
<tr>
<th>Algo. Parameters</th>
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</thead>
<tbody>
<tr>
<td>Iteration=2000</td>
<td>Iteration=2000</td>
</tr>
<tr>
<td>Population=500</td>
<td>Nests=25</td>
</tr>
<tr>
<td>α=1.05; β=1.1</td>
<td>pα = 0.25</td>
</tr>
<tr>
<td>FA</td>
<td>TPO</td>
</tr>
<tr>
<td>Iteration=2000</td>
<td>Iteration=2000</td>
</tr>
<tr>
<td>Fireflies=20</td>
<td>Leaves=500</td>
</tr>
<tr>
<td>α = 0.25; β = 0.2; γ = 1</td>
<td>Branch=20</td>
</tr>
<tr>
<td></td>
<td>α = 0.3; β = 0.7; θ = 0.01</td>
</tr>
</tbody>
</table>

Figure 1. Engineering design problems with (a): Three-bar truss [16], (b): Spring design problem [17] and (c) Golinski speed reducer problem [16], [17]

### 4. Results and Analysis

The comparison between each algorithm is executed by 2.6GHz computer processor. The simulation and statistical analysis are carried out using MATLAB and STATSGRAPHICS Centurion respectively. Each algorithm is executed ten times for each problem and the obtained results are evaluated. The focuses of evaluation include statistical results and convergence.

#### 4.1. Statistical Comparison

The statistical results of each algorithm by every problem are tabulated in Table 4. The best result of each category is highlighted in bold. The values in Table 4 are divided by constrained problem (F), average, best value, worst value and standard deviation (σ). Based on the results, PSO has the lowest average and the best solution in F1 followed by CS and TPO. TPO has the lowest variation among the results of F1. In F2 optimization, TPO shows the best results for all categories followed by FA, PSO and CS for the mean value. In F3, CS has the lowest results for all categories followed by TPO. TPO also outperform other algorithms in F4 with lowest average and variation of the results. Overall, most of the best results are shown by TPO and CS. The advantage of TPO is due to the parallel search of leaves in each defined branches, which is equivalent to (leaves x branches) search agents. Thus the search space is broader within the constrained area.
4.2. Convergence Analysis

The convergence of each algorithm in single run is shown in Figure 2. The figure is divided into four charts that represent each constrained problem (F1–F4). Based on the figure, TPO has fastest convergence towards global optimum in all four constrained problems followed by CS (in F2 and F4) and FA (in F1 and F3). PSO is not able to converge in F3 and F4 as also shown in Table 4. The complexity for global search increases from F1 to F4 as can be observed in F1, whereas all algorithms successfully converged to global optimum in 200th iteration. However, some algorithm starts to detoriate and converge towards global optimum slower compared to others.

Figure 2. Convergence of best results for each algorithm with (a) three-bar truss, (b) spring design, (c) speed reducer and (d) Himmelblau’s nonlinear function
5. Conclusion

This paper proposes a novel Tree Physiology Optimization to solve constrained optimization problem. The performance of proposed algorithm is compared with other existing algorithms on three engineering design problems and a mathematical nonlinear optimization problem. The performance measures for the comparison include statistical results for each optimization problem and convergence towards global optimum solution. In statistical results, TPO has the best mean value for F1 and F4, PSO has the lowest average for F2 and CS for F3. TPO also has the lowest standard deviation in F1,F2 and F4. The advantage of parallel search of TPO from individual leaves and branches as search agents resulted in broader search and thus faster convergence and finding the global optimum. Due to the parallel search (leaves x branches), there is a higher chance to find the current global optimum in each iteration. Therefore the standard deviation of solution in TPO search is smaller. For CS algorithm, the advantage of levy flight led to longer exploration step length in the long run.

With Levy flight, the exploitation in local search is also faster compared to normal random walk. These properties is observed in statistical results in F3 and some other problem types that shows better value compared to several other algorithm. The convergence of CS also show dispersed solution in the beginning of iteration, but then able to exploit the global optimum after some iteration elapsed. FA algorithm has fast convergence in the problem with lower variable dimension. The convergence for higher dimension variables of FA algorithm can be improved further by increasing the number of fireflies. The PSO algorithm has the lowest global optimum in low dimension problem. Nonetheless, its stability problems restrict the success rate as the performance decreases by higher number of dimension variables. The finding imply more similarity studies in diverse fields particularly in real world problem since such problems correspond the the advance of science and technology which may consists numerous constraints that need to be considered. Several paradigms that need be considered are construction engineering, manufacturing and control technology.

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References


