Field Distribution of Environment and Vibration Chamber

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Abstract

Environment and vibration test system can test equipment such as radars in full load condition. Environment and vibration chamber is an opening rectangular cavity. In order to study the field distribution of environment and vibration chamber, the open rectangular cavity field distribution and parabolic antenna are analyzed by using the Dyadic Green’s functions. According to the Dyadic Green's functions, the field distribution of environment and vibration chamber is concluded by MATLAB. The simulation result of FEKO proves that the method by using Dyadic Green’s functions is available. The innovation of this article is using the Dyadic Green’s functions to analyzed open cavity field and parabolic antenna.

Key worlds: Dyadic Green’s functions, environment and vibration chamber, FEKO, MATLAB

1. Introduction

Environment and vibration test system can test equipment under test such as radars in full load condition. The system includes environment and vibration chamber and microwave chamber. The system could overall accurately reflect equipment under test such as radars performance in high temperature, high humidity and strong vibration environment, could still find temporary failure of equipment under test. But in the experimental process find that the microwave of leakage strengthened because of multiple reflections in metal environment and vibration chamber, the energy is strong enough to influence radars performance, or even damage radars. Excessive leakage microwave could affect the accuracy of equipment under test, so field distribution of environment and vibration chamber are analyzed to find the solution.

In recent years, due to the reverberation chamber in the application of electromagnetic compatibility [1], the field distribution of rectangular cavity is mostly analyzed by Dyadic Green’s functions [1, 2]. However, related researches of field distribution of open rectangular cavity were rare. In reverberation chamber study, generally use line antennas as source, and then source can be simplified for current source [3]. Few people involved in researches of parabolic antenna in rectangular cavity. This paper, open rectangular cavity and parabolic antennas of rectangular cavity was discussed. And prove the correctness of this method in this paper with the test results.

2. Dyadic Green’s Functions in Rectangular Cavity

Environment and vibration chamber is a front opening rectangular cavity. Its structure is shown in Figure 1.

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In Figure 1, the circular in the chamber is parabolic antenna of equipment under test. The front opening of the chamber is facing microwave chamber, size as shown in Figure 1. Environment and vibration chamber is integrated the temperature, humidity and vibration three functions. In the chamber, temperature and humidity can be controlled. At the same time interior also contains a vibration test bench that could simulate vibration condition of the airborne radar in operation process. From Figure 1, environment and vibration chamber is a front opening rectangular cavity.

Figure 2 is completely closed rectangular cavity. Elliptic area is current source. Establish coordinate system as shown in Figure 2, l, a, b are chamber length, width and height.

Make the wave equation into the green functions form [4,5], that is
\[ \nabla \times \nabla \times G_s(R, R') - k^2 G_s(R, R') = i \delta(R - R') \]. The pure conductor of the rectangular cavity boundary conditions is \[ n \times G_s(R, R') = 0 \] (in the border, that is \( x = 0, x = a \) and \( y = 0, y = b \) and \( z = 0, z = l \)) . And then get Dyadic Green’s functions in rectangular chamber [6,7]:

\[
G_s(R, R') = -\frac{1}{k^2} \frac{\delta}{\delta z} \delta(R - R') \\
+ \frac{2}{ab} \sum_{m,n} \frac{(2-\delta_0)}{k_x^2 k_x \sin k_x l} \\
\left[ M_{co}^l(l-z) M_{co}'^o(z') \right] \quad (z > z') \\
- \frac{2}{ab} \sum_{m,n} \frac{(2-\delta_0)}{k_x^2 k_x \sin k_x l} \\
\left[ M_{co}^o(z) M_{co}'^o(l-z') \right] \quad (z < z') \\
\] (1)

where:

\[
M_{co}^o(l-z) = \sin(k_y b) \quad M_{co}^o(k_y) \\
-e^{ik_z l} M_{co}^o(z) \\
\] (2)
\[ N_{\omega}(l-z) = -j \sin(k_z l) N_{\omega}(k_z) \]
\[ + e^{\beta l} N_{\omega}(z) \]  
(3)

\[ M_{\omega}(z) = \frac{1}{2} [M_{\omega}(-k_z) - M_{\omega}(k_z)] \]  
(4)

\[ N_{\omega}(z) = \frac{1}{2} [N_{\omega}(-k_z) + N_{\omega}(k_z)] \]  
(5)

\[ M_{\omega}(k_z) = (-k_z C_x S_y \hat{x} + k_z S_x C_y \hat{y}) e^{\beta z} \]  
(6)

\[ N_{\omega}(k_z) = \frac{1}{k} (jk_z k_x C_x S_y \hat{x} + \hat{z} + k_z^2 C_x S_y \hat{z}) e^{jk_z} \]  
(7)

\[ \hat{x}, \hat{y}, \hat{z} \] are respectively for unit vector on the direction of \( x, y, z \), \( k_x = \frac{m \pi}{a} \); \( k_y = \frac{m \pi}{b} \); \( S_x = \sin k_x x \); \( S_y = \sin k_y y \); \( C_x = \cos k_x x \); \( C_y = \cos k_y y \); \( k^2 = k_x^2 + k_y^2 + k_z^2 \);

\[ k_c^2 = k_x^2 + k_y^2 \); \( \delta_0 \) is Kronecker delta. When \( m = 0 \) or \( n = 0 \), \( \delta_0 = 1 \). When \( m \neq 0 \) and \( n \neq 0 \), \( \delta_0 = 0 \) [7].

Eq. (1) to Eq. (7) gives active rectangular cavity in Dyadic Green’s functions. The relationship of field strength and \( \overline{G}_x(R, R') \) will be discussed in this paper. With the relation between them, electric field strength can be obtained.

### 3. Dyadic Green’s Functions in Open Rectangular Cavity

The structure of the open rectangular cavity as shown in Figure 3, establish coordinate system as shown.

In general, using Maxwell’s equations in an active area, the Dyadic Green’s function of electromagnetic wave equation is:

\[ \nabla \times \nabla \times \overline{G}_x(R, R') - k^2 \overline{G}_x(R, R') = \tilde{I} \delta(R - R') \]  
(8)

Where: \( k = 2 \pi / \lambda \) is wave number. Using the second vector-Dyadic Green’s theorem:

\[ \iiint \left( \nabla \times (\nabla \times \overline{Q} - (\nabla \times \overline{P}) \times \overline{Q}) \right) dV = -\iiint \tilde{n} \left( \nabla \times (\nabla \times \overline{P}) \times \overline{Q} \right) dS \]  
(9)

By applying \( \overline{Q} = \overline{G}_x(R, R') \) and \( \overline{P} = \overline{E}(R) \) into Eq. (9), Eq. (9) will be:

\[ \overline{E}(R) - j\omega \mu \iiint \overline{J}(R) \overline{G}_x(R, R') dV = -\iiint \tilde{n} \left( \nabla \times \overline{E}(R) \times \nabla \times \overline{G}_x(R, R') \right) dS \]  
(10)
By applying \( \nabla \times \mathbf{E}(\mathbf{R}) = j \omega \mu_0 \mathbf{H}(\mathbf{R}) \) and \( a \times (b \times c) = -b \cdot (a \times c) = (b \times a) \times c \) into Eq. (10), Eq. (10) will be:

\[
\begin{align*}
E(\mathbf{R}') &- j \omega \mu_0 \iint \mathbf{J}(\mathbf{R}) \overline{G}_r(\mathbf{R}, \mathbf{R}') dV \\
&= \int \left( [\hat{n} \times \mathbf{E}(\mathbf{R})] \nabla \times \overline{G}_r(\mathbf{R}, \mathbf{R}') \right) dS \\
&+ \int \left( [\hat{n} \times \mathbf{E}(\mathbf{R})] \nabla \times \overline{G}_r(\mathbf{R}, \mathbf{R}') \right) dS
\end{align*}
\]

In Eq. (11), \( \mathbf{R}' \) is position vector of field, \( \mathbf{R} \) is position vector of source. Under the condition of open rectangular cavity, surface integral can be divided into two \( S_B \), \( S_B \) and \( S_K \), \( S_B \) is surface of the conductor wall, \( S_K \) is the open face. In the surface of the cavity, boundary condition is: \( \hat{n} \times \overline{G}_r(\mathbf{R}, \mathbf{R}') = 0 \). In Eq. (11), the direction of wave is \( \hat{n} \) satisfies that \( \hat{n} = z \). Then, Eq. (11) can deduce:

\[
\begin{align*}
E(\mathbf{R}') &- j \omega \mu_0 \iint \mathbf{J}(\mathbf{R}) \overline{G}_r(\mathbf{R}, \mathbf{R}') dV \\
&= -\int \left( [\hat{n} \times \mathbf{E}(\mathbf{R})] \nabla \times \overline{G}_r(\mathbf{R}, \mathbf{R}') \right) dS \\
&+ -\int \left( [\hat{n} \times \mathbf{E}(\mathbf{R})] \nabla \times \overline{G}_r(\mathbf{R}, \mathbf{R}') \right) dS
\end{align*}
\]

Eq. (13) is the field distribution of open rectangular cavity of Dyadic Green's function.

4. Parabolic Antenna in Open Rectangular Cavity

Because the radiation antenna of radar is parabolic antenna, the parabolic antenna is source, then there is no current source in rectangular cavity [8, 9]. So surface integral of Eq. (11) can be divided into two parts, one part is surface integral of paraboloid \( S_P \), the other part is surface integral of surface of the rectangular cavity \( S_B \). The property of first kind of electricity Dyadic Green's function: \( \hat{n} \times \overline{G}_r(\mathbf{R}, \mathbf{R}') = 0 \). Eq. (11) can deduce:

\[
\begin{align*}
E(\mathbf{R}') &- j \omega \mu_0 \iint \mathbf{J}(\mathbf{R}) \overline{G}_r(\mathbf{R}, \mathbf{R}') dV \\
&- -\int \left( [\hat{n} \times \mathbf{E}(\mathbf{R})] \nabla \times \overline{G}_r(\mathbf{R}, \mathbf{R}') \right) dS \\
&- -\int \left( [\hat{n} \times \mathbf{E}(\mathbf{R})] \nabla \times \overline{G}_r(\mathbf{R}, \mathbf{R}') \right) dS
\end{align*}
\]

Because of pure conductor wall of the cavity, on the surface of \( S_B \), the boundary condition is \( \hat{n} \times \mathbf{E}(\mathbf{R}) = 0 \), Eq. (14) can deduce:
\[ E(R') = -\iint_{S_p} \left[ \hat{n} \times E(R) \right] \] 
\[ \times \nabla \times G_e(R, R') \] 
\[ dS \] 
(15)

Where paraboloid of parabolic antenna is \( S_p \), \( S_K \) is opening of environment and vibration chamber. The first part of surface integral \( E(R') \) is field strength vector of paraboloid surface. \( E(R') \) is field strength vector of solving point. Equivalent field strength of paraboloid surface is known [10]:

\[ E_i(\varphi, \xi) = \frac{\sqrt{60P_D(\varphi, \xi)}}{\rho} \] 
(16)

Figure 4 is the side section of parabolic antenna. Where point \( F \) is focus of parabolic antenna, angle \( \varphi \) is the angle between line from \( M \) any point of parabolic antenna to \( F \) and \( Z \) axis.

Point \( M' \) in Figure 5 is the projection of \( M \) on the caliber face. The distance between \( M' \) and the centre of the caliber face is \( R \), and angle between the line from \( M' \) to the centre of the caliber face and diameter of the caliber face is \( \xi \).

In Eq. (16), \( P_r \) is total radiation power, \( D_j(\varphi, \xi) \) is direction coefficient, where

\[ D_j(\varphi, \xi) = \frac{4\pi}{\lambda^2} S \nu \] 
\[ S = \pi R_0^2 = 4\pi f^2 \tan^2 \frac{\varphi_0}{2} \] 
\[ \nu = 2 \cot \frac{\varphi_0}{2} \left[ \int_0^\varphi_0 \cos^{n/2} \varphi \tan \varphi \frac{d\varphi}{2} \right] \left[ \int_0^\varphi_0 \cos^{n''} \varphi \sin \varphi d\varphi \right] \]

In order to facilitate calculation, we can use the function \( x^2 + y^2 = 4fz \) to deduce Eq. (16). Substitute \( \tan \varphi = \frac{\sqrt{x^2 + y^2}}{f - z} \) and \( \rho = \sqrt{x^2 + y^2 + (f - z)^2} \) into Eq. (16):

\[ E_i(x, y, z) = \frac{4\pi f \tan \frac{\varphi_0}{2} \sqrt{60P_D \nu}}{\lambda \sqrt{x^2 + y^2 + (f - z)^2}} \] 
(17)
Except unknown quantities \( x, y, z \), the others are all known. Once Dyadic Green’s function in the rectangular cavity through Eq. (1) is got, we also gain field strength of any point in the cavity using Eq. (15).

5. The Data Analysis

In order to verify the accuracy of the proposed method, the Figure of environment and vibration chamber field distribution is concluded by MATLAB. At the same time, the result of field distribution in the chamber is got by FEKO. Comparing the two results, the method in this paper can be proved.

In the course of calculation, the radius of parabolic antenna is 30cm, the frequency is 1GHz. The results are shown in Figure 6 to Figure 9.

![Figure 6. Field distribution in 1.8m away from origin of the chamber by MATLAB](image)

![Figure 7. Field distribution in 1.8m away from origin of the chamber by FEKO](image)

![Figure 8. Field distribution in 2.0m away from origin of the chamber by MATLAB](image)

![Figure 9. Field distribution in 2.0m away from origin of the chamber by FEKO](image)

Figure 6 is the electric field strength distribution in 1.8m away from origin of the chamber by MATLAB. Compare with Figure 7 that is the simulation result of FEKO in the same position. The two results both compos of peaks, the two results is similar. The average of peak values of the field distribution in 1.8m away from origin of the chamber is 1900V/m by MATLAB. And the average value by FEKO is 1679V/m. By MATLAB the average of field distribution is 969V/m. By FEKO average value is 898V/m. That means field distribution of two methods of calculating is approximately equal.

The method in this paper is used in MATLAB, but the method used in FEKO is moment method. That is a reason of error. FEKO is an electromagnetic simulation software package. Therefore there are some limitations for input model. Using Dyadic Green’s functions could Optional change parameters by MATLAB; the calculation results would be more precise. That is to say that using Dyadic Green’s functions are more flexible. What’s more, moment method
could meet variety models, is no advantage in rectangular cavity. Dyadic Green's functions are mainly aimed at rectangular cavity. Using method in this paper will get the more accurate result.

6. Conclusion
In this paper, environment and vibration chamber field distribution is analyzed by the application of Dyadic Green's functions. Because of Dyadic Green's functions using the vector, there are large amount of data. Some simplified methods were used to reduce the amount of data in calculation, thus may cause some calculation errors. Through the experiment data, the results of MATLAB and the simulation results of FEKO are approximate. The correctness of this paper is proved. In addition, using Dyadic Green's functions in MATLAB is more flexible, parameter settings are more direct, and calculation process is visible. All these advantages are FEKO doesn't have.

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