Numerical Simulation of Highly-Nonlinear Dispersion-Shifted Fiber Optical Parametric Gain Spectrum with Fiber Loss and Higher-Order Dispersion Coefficients

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Abstract
The previous study investigated the fiber parameters on the analytical one-pump fiber optical parametric amplifier (FOPA) gain spectrum of a loss-free highly-nonlinear dispersion-shifted fiber (HNL-DSF) and got the optimum results for each parameter. However, the FOPA gain of the combination of all the optimum values of the considered parameters was not reported. Hence, this paper intends to investigate the analytical FOPA gain of the combination of all the optimum values of the considered parameters from the previous study. Later, the analytical gain was compared with the numerical gain from the fourth-order Runge-Kutta (RK4) method and Matlab built-in function, ode45. Next, the effect of fiber loss and higher order dispersion coefficients such as fourth and sixth-order dispersion coefficients were studied. It is found that RK4 gives a smaller error and the gain reduces while the bandwidth remains same in presence of fiber loss. The fourth-order dispersion coefficient broadens the bandwidth a bit while maintaining the gain and there are two narrow-band gains generated to the left and right-side of the broad-band gain. The sixth-order dispersion coefficient just shifts the two narrow-band gains toward or away from the broadband gain depending on the positive or negative signs of the sixth-order dispersion coefficient.

Keywords: HNL-DSF, FOPA, RK4, ode45, higher-order dispersion coefficients

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1. Introduction
Optical communication is used in handling the ever increasing demand of Internet traffic. The propagation of an optical pulse through optical fiber at long haul distance will experience the decay in amplitude because the nonlinearity cannot compensate the dispersion due to the fiber loss. Hence, the optical pulse needs to be amplified. The commonly used amplifier is Erbium-doped fiber amplifier (EDFA) which was invented in 1980. It amplifies light in C-band (wavelength ranges 1530nm-1565nm) where telecommunication fibers have a minimum loss. However, EDFA adds noise and nonlinearity to the amplified signal. Another family of the optical amplifier is Raman amplifier (RA) which uses stimulated Raman scattering (SRS) to transfer energy from one or more pump sources to the transmitted optical signal. Although, Raman amplification can operate at any wavelength depending on the pump wavelength, but, it needs high pump power and has problems with double Rayleigh backscattering (DRB) which creates noise.

Fiber optical parametric amplifier (FOPA) is operated based on four-wave mixing (FWM) process. FWM occurs when a signal light with angular frequency $\omega_s$ is launched in an optical fiber along with two strong pumps with angular frequencies $\omega_{p1}$ and $\omega_{p2}$. At the end of the fiber, the signal is amplified and an idler wave is generated with angular frequency $\omega_i$ through phase-matching condition, $\omega_i = \omega_{p1} + \omega_{p2} - \omega_s$. When the frequencies of these two pumping waves are identical, the "degenerated four-wave mixing" (DFWM) occurs such that $\omega_i = 2\omega_p - \omega_s$. FOPAs can be classified into one-pump (1-P) where a single pump wave is used in fiber and into two pump (2-P)
in which two pump waves are applied into the fiber. FOPA can perform similar functions comparable to existing amplifiers on top of remarkable features that are not offered by existing amplifiers such as wide bandwidth [1], adjustable gain spectra, adjustable center frequency, wavelength conversion, phase conjugation, pulsed operation for signal processing and 0-dB noise figure [2].

The development of a dispersion-shifted fiber (DSF) with zero-dispersion wavelength (ZDW) located in the C-band [3] and the same of a silica-based highly-nonlinear DSF (HNL-DSF) [4] has sparked the researches of FOPA. A good FOPA needs to provide broad bandwidth and high gain. The performance of a FOPA is dependent on fiber parameters. The influence of fiber parameters such as fiber length, pump power and dispersion order towards a host lead-silicate based binary uni-clad microstructure fiber was analyzed by [5]. Cheng et. al [6] studied the effect of pump wavelengths and powers on dual-pump configuration of FWM on highly nonlinear tellurite fibers with tailored chromatic dispersion. Maji and Chaudhuri [7] investigated fiber parameters such as fiber length, pump power and pump wavelength towards the analytical FOPA gains of three ZDW in the near-zero ultra-flat photonic crystal fibers (PCF) around the communication wavelength by tuning pumping condition. A study in [8] concluded that by changing temperature of the fiber and tuning the chromatic dispersion profile and ZDW of the optimized PCF, then a wide gain spectrum in the communication wavelength can be achieved.

Othman et.al [9] studied the influence of fiber parameters like fiber length, pump power, pump wavelength and dispersion slope on the analytical FOPA gain of a single-pump loss-free HNL-DSF. They showed that the long fiber length, the high pump power, the pump wavelength close to ZDW and the small dispersion slope give a wider bandwidth and a high gain. However, they did not plot the combination of the optimum cases for each of the above-mentioned parameters and they considered HNL-DSF as lossless.

Pakarzadeh and Bagheri [10] investigated the gain spectrum and saturation behavior of one-pump FOPA in the presence of the fourth-order dispersion coefficient. They concluded that when the pump wavelength is near to or exactly at ZDW and when the difference between the signal and pump waves is large enough, the fourth-order coefficient $\beta_4$ is important to be taken into account. Dainese [11] presented a scheme to optimize the PCF chromatic dispersion curve and higher order dispersion coefficients up to $\beta_{\text{bd}}$ for a broadband FOPA.

Motivated with the work of [9] and the importance to include higher order dispersion when the pump wavelength close to or exactly at ZDW and when the difference between the signal and pump waves is large enough. Hence, hence in this paper, the optimum combination from [9] is utilized and the analytical gain is compared with the fourth-order Runge-Kutta (RK4) method and with Matlab built-in ordinary differential equation (ode) function, which is ode45. Practically, fiber is not loss-free for which the fiber loss is added to the optimum combination case and the problem is solved by using the RK4 method. Lastly, the higher dispersion coefficients $\beta_4$ and $\beta_6$ are added slowly to see their effects on the FOPA performance.

2. Mathematical Model

The three coupled amplitude equations for pump $A_p$, signal $A_s$, and idler $A_i$, that describe the propagation of three interacting waves in 1-P FOPA are given by [12] as

$$\frac{dA_p}{dz} = i\gamma \left[ A_p \left( |A_p|^2 + 2(|A_s|^2 + |A_i|^2) \right) + 2A_s A_i A_p^* \exp(i\Delta \beta z) \right] - \frac{1}{2} \alpha A_p = f_1(z, A_p, A_s, A_i),$$

$$\frac{dA_s}{dz} = i\gamma \left[ A_s \left( |A_s|^2 + 2(|A_p|^2 + |A_i|^2) \right) + A_p^* A_i A_s^* \exp(-i\Delta \beta z) \right] - \frac{1}{2} \alpha A_s = f_2(z, A_p, A_s, A_i),$$

$$\frac{dA_i}{dz} = i\gamma \left[ A_i \left( |A_i|^2 + 2(|A_p|^2 + |A_s|^2) \right) + A_p A_s A_i^* \exp(-i\Delta \beta z) \right] - \frac{1}{2} \alpha A_i = f_3(z, A_p, A_s, A_i).$$

The first term of the right-hand side of the equations represents the self-phase modulation (SPM), whereas the second and third terms are cross-phase modulation (XPM). The fourth term acts as power transfer due to FWM and fiber attenuation is represented at the last term.

Here $\alpha$ is the fiber loss, $\gamma$ is the nonlinear parameter and $^*$ represents complex conjugate. Meanwhile, $\Delta \beta$ is the linear wave vector mismatch between the interacting waves of pump, signal
and idler as given by [12]
\[
\Delta \beta = 2 \sum_{m=1}^{\infty} \frac{\beta_{2m}}{(2m)!} (\Delta \omega)^{2m},
\]
where \(\Delta \omega = \omega_s - \omega_p\) is the frequency detuning, \(\omega_k = 2\pi c/\lambda_k\) for \(k \in \{p, s\}\) are the frequencies of pump and signal in terms of wavelength, \(\lambda_k\). The dispersion coefficients of odd order do not contribute to the gain spectrum and hence only even order coefficients are taken into account [12]. By taking \(m = 1, 2\) and \(3\) in expression (4), the linear wave vector mismatch up to the sixth order dispersion coefficient \(\beta_6\) is expressed as
\[
\Delta \beta = \beta_2 (\Delta \omega)^2 + \frac{\beta_4}{12} (\Delta \omega)^4 + \frac{\beta_6}{360} (\Delta \omega)^6.
\]
For \(m = 1, 2\) and \(3\), \(\beta_2, \beta_4\) and \(\beta_6\) are illustrated respectively by [5] as
\[
\beta_{2m} = \sum_{n=2m}^{\infty} \frac{\beta_{n0}}{(n - 2m)!} (\omega_p - \omega_0)^{n-2m},
\]
where \(\beta_{n0} = d^n \beta(\omega)/d\omega^n|_{\omega=\omega_0}\) is the dispersion coefficients calculated at the zero dispersion frequency, \(\omega_0\). By combining (5) and (6) for \(m = 1\) to \(3\), the \(\Delta \beta\) can be written as
\[
\Delta \beta = \beta_2 (\Delta \omega)^2 + \frac{\beta_4}{3} \left[ 2(\omega_p - \omega_0)^2 + \frac{1}{4} (\Delta \omega)^2 \right] (\Delta \omega)^2 + \frac{\beta_6}{24} \left[ (\omega_p - \omega_0)^4 + \frac{1}{15} (\Delta \omega)^4 \right] (\Delta \omega)^2.
\]
Here \(\beta_2, \beta_4\) and \(\beta_6\) are the second, fourth and sixth-order dispersion constants. Eq. (7) shows the significance of \(\beta_4\) and \(\beta_6\) when pump wavelength approaches to or precisely at ZDW and \(\Delta \omega\) is large.

If the fiber loss is negligible in Eqs (1)-(3), the analytical parametric gain is written as [13]
\[
G = 1 + \left[ \frac{\gamma P_p}{g \sinh(gL)} \right]^2,
\]
with \(L\) as fiber length, \(P_p\) as pump power and \(g\) is expressed as
\[
g = \sqrt{(\gamma P_p)^2 - \frac{\kappa^2}{2}}
\]
and the total phase-mismatch, \(\kappa\) is indicated by
\[
\kappa = \Delta \beta + 2\gamma P_p.
\]

3. Fourth-Order Runge-Kutta (RK4) Method

If the fiber loss is included, the Eqs. (1)-(3) cannot be solved analytically to obtain the analytical gain. Hence the spatial variable \(z\) in Eqs. (1)-(3) is made discrete into \(n\) segments with a step size \(h\) such that \(z^0 = 0, z^1 = z^0 + h, z^2 = z^1 + h, \ldots, z^n = z^{n-1} + h = L\). With \(j = 0\) and initial amplitudes for pump \(A_p^0\), signal \(A_s^0\) and idler \(A_i^0\), at \(z_0 = 0\), the amplitudes for pump \(A_p^1\), signal \(A_s^1\) and idler \(A_i^1\) at next spatial \(z^1\) are given by the RK4 method as follows:
\[
\begin{align*}
A_p^{j+1} &= A_p^j + \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}, \\
A_s^{j+1} &= A_s^j + \frac{l_1 + 2l_2 + 2l_3 + l_4}{6}, \\
A_i^{j+1} &= A_i^j + \frac{m_1 + 2m_2 + 2m_3 + m_4}{6}, \quad \text{for} \quad j = 0, 1, \ldots, n
\end{align*}
\]
Where
\[
\begin{align*}
 k_1 &= hf_1(z, A_p, A_s, A_i), \\
 k_2 &= hf_1 \left( z + \frac{h}{2}, A_p + \frac{k_2}{2} A_s + \frac{l_2}{2}, A_i + \frac{m_2}{2} \right), \\
 k_3 &= hf_1 \left( z + \frac{h}{2}, A_p + \frac{k_2}{2} A_s + \frac{l_2}{2}, A_i + \frac{m_2}{2} \right), \\
 k_4 &= hf_1(z + h, A_p + k_3, A_s + l_3, A_i + m_3), \\
 l_1 &= hf_2(z, A_p, A_s, A_i), \\
 l_2 &= hf_2 \left( z + \frac{h}{2}, A_p + \frac{k_2}{2} A_s + \frac{l_2}{2}, A_i + \frac{m_2}{2} \right), \\
 l_3 &= hf_2 \left( z + \frac{h}{2}, A_p + \frac{k_2}{2} A_s + \frac{l_2}{2}, A_i + \frac{m_2}{2} \right), \\
 l_4 &= hf_2(z + h, A_p + k_3, A_s + l_3, A_i + m_3), \\
 m_1 &= hf_3(z, A_p, A_s, A_i), \\
 m_2 &= hf_3 \left( z + \frac{h}{2}, A_p + \frac{k_2}{2} A_s + \frac{l_2}{2}, A_i + \frac{m_2}{2} \right), \\
 m_3 &= hf_3 \left( z + \frac{h}{2}, A_p + \frac{k_2}{2} A_s + \frac{l_2}{2}, A_i + \frac{m_2}{2} \right),
\end{align*}
\]
(14)

With amplitudes for pump \( A_p^{1} \), signal \( A_s^{1} \) and idler \( A_i^{1} \) at spatial \( z^{1} \), the amplitudes for pump \( A_p^{2} \), signal \( A_s^{2} \) and idler \( A_i^{2} \) at next spatial \( z^{2} \) is calculated iteratively from Eq. (13). The process is iterated till \( z^{3} = z^{n} = L \). At the end of the fiber, let the output signal amplitude \( A_{s, out}^{n} = A_{s, out} \). The numerical gain \( G \) is calculated as \( G_{\text{numerical}} = 10 \log(P_{s, out}/P_{s, in}) \), where \( P_{s, out} \) and \( P_{s, in} \) are output and input signal powers respectively. \( P_{s, out} = |A_{s, out}|^{2} \) while \( P_{s, in} = |A_{s, in}|^{2} \).

4. Specification of HNL-DSF

In this numerical simulation, a HNL-DSF of OFS company with \( \alpha = 0.82 dB/km, \gamma = 11.5 W^{-1} km^{-1} \) and ZDW at \( \lambda_0 = 1556.5 nm \) were used as the parametric gain medium. Then, the optimum case of fiber length \( L = 500 m \), pump power, \( P_p = 30 dBm \), pump wavelength, \( \lambda_p = 1559 \)nm and dispersion slope \( s = 0.01 \) from [9] were used in this simulation with signal wavelength from 1400nm to 1700nm. Next, the dispersion \( D \) from the fiber data sheet versus wavelength was plotted. Subsequently, the second-order dispersion coefficient, \( \beta_2 \) was calculated from
\[
\beta_2 = -\frac{\lambda^2}{2\pi c} D,
\]
(15)
where \( c \) is speed of light.

After that, a second-order degree polynomial fit, \( D = a\lambda^2 + b\lambda + d \), where \( a, b \) and \( d \) are some coefficients was performed to obtain the quadratic equation for \( D \). Later, \( D \) was differentiate twice to get a first-order derivative, \( D_\lambda \) and second-order derivatives, \( \frac{dD_\lambda}{d\lambda} \) in order to obtain the fourth-order and sixth-order dispersion coefficients from the following respective equations:
\[
\begin{align*}
 \beta_4 &= -\frac{\lambda^4}{(2\pi c)^3} (6D + 6\lambda D_\lambda + \lambda^2 \frac{dD_\lambda}{d\lambda}), \\
 \beta_6 &= -\frac{\lambda^6}{(2\pi c)^5} (120D + 240\lambda D_\lambda + 120\lambda^2 \frac{dD_\lambda}{d\lambda} + 20\lambda^3 \frac{d^2D_\lambda}{d\lambda^2} + \lambda^4 \frac{d^3D_\lambda}{d\lambda^3}).
\end{align*}
\]
(16)
(17)
The second, fourth and sixth-order dispersion coefficients were obtained as \( \beta_2 = -3.872 \times 10^{-2} ps^2/km \), \( \beta_4 = 6.327 \times 10^{-5} ps^4/km \) and \( \beta_6 = 1.186 \times 10^{-8} ps^6/km \).

5. Results and Analysis
The analytical gain of a lossless HNL-DSF was calculated by using Eqs.(8)-(10). Figure 1(a) portrays the analytical gain of the above lossless HNL-DSF. It shows a gain spectrum from 1514nm to 1607nm with a peak gain of 43.9234dB which has a broader bandwidth if compared to
Meanwhile, Eqs (1)-(3) with $\alpha = 0\, dB/km$ were solved numerically by using Eqs. (11)-(13) of the RK4 method with a step size $h = 0.1$. The RK4 gain was plotted in Fig. 1(b). Intuitively, both Figs. 1(a) and 1(b) are indistinguishable.

Later, Eqs.(1)-(3) with $\alpha = 0dB/km$ were again solved numerically by using Matlab built-in ode function, known as ode45. The ode45 gain was plotted in Fig. 2(b). Fig. 2(a) depicts the analytical gain. Here also, both the Figs. 2(a) and 2(b) are identical.

To check whether both the analytical gain (Fig. 1(a)) and RK4 gain (Fig. 1(b)) are absolutely similar, they were plotted on the same graph as displayed in Fig. 3(a). Whereas, both the analytical gain (Fig. 2(a)) and ode45 gain (Fig. 2(b)) were plotted on the same graph as displayed in Fig. 3(b) to check if they are literally look-alike. It is noticed that both the analytical and RK4 gains are overlapped accurately. Similarly, both the analytical and ode45 gains are overlapped on one another.
Even though graphically both the analytical and RK4 gains are overlapped exactly, the absolute errors between the analytical and RK4 gains were calculated and plotted in Fig. 4(a) which reveals that still there are some small errors between the analytical and RK4 gains with the maximum error of 0.02215. On the other hand, the absolute errors between the analytical and ode45 gains were pictured in Fig. 4(b) and it exposes that there are some errors between the analytical and ode45 gains with a maximum error of 0.9682. Comparatively, Fig 4 proves that the RK4 method gives a smaller error if compared to ode45.

Furthermore, there is no analytical solution if fiber loss as $\alpha = 0.82\, dB/km$ is taken into consideration in Eqs.(1)-(3). Hence, based on the smaller error given by the RK4 method in comparison with ode45 in Fig. 4, Eqs.(1)-(3) were solved numerically by the RK4 method up to the second order dispersion coefficient as expressed in vector mismatch Eq. (7). The FOPA gains for both the loss-free fiber and lossy fiber with an attenuation coefficient of $\alpha = 0.82\, dB/km$
are shown in Fig. 5. It is found that the peak gain is reduced when the fiber loss is taken into account, but the bandwidth remains the same. This pattern is similar with research work in [10] that included fiber loss and the fourth-order coefficient. They compared their work with previous research works without including fiber loss and the fourth-order coefficient and it was observed that the peak gain spectrum with the fiber loss was reduced.

Lastly, to study the effect of higher-order dispersion coefficient such as the fourth order and sixth order dispersion coefficients toward the performance of a lossy HNL-DSF FOPA, Eqs.(1-3) were solved numerically by the RK4 method up to the fourth order and sixth order dispersion coefficients given in vector mismatch Eq. (7) one by one. The FOPA gains of the second, fourth and sixth order dispersion coefficients are given in Fig. 6. It is noticed that, the fourth-order dispersion coefficient broadens the bandwidth a bit while maintaining the gain peak as reported by [10]. Besides, there are two narrow-band gains at the left and right sides of the wide-band gain due to the effect of the fourth-order dispersion coefficient. The narrow-band at the right side of the wide-band gain was reported by [11] and [14] in their half range FOPA gain spectrum when the negative sign of the fourth-order dispersion coefficient was present. It is observed that the positive sign of the sixth order dispersion coefficient just moves the two narrow-band gain toward the wide-band gain while the peak gain is unchanged and its broadband gain is overlapped exactly with the broadband gain from the fourth-order dispersion coefficient.

If the sixth-order dispersion coefficient takes negative sign as seen in Fig. 7, the two
narrow-band gains will move away from the wide-band gain in comparison to the positive sign of the sixth-order dispersion coefficient. The broadband gain of the negative sign of the sixth order dispersion coefficient is overlapped exactly with the broadband gain from the fourth-order dispersion coefficient too.

Figure 7. The gain spectrum up to second, fourth and negative sixth-order dispersion coefficient of a lossy HNL-DSF.

6. Conclusion

This paper has simulated the analytical gain spectrum of the combination of all the optimum values of a lossless HNL-DSF parameters [9] and compared with the RK4 method as well as Matlab built-in function ode45. It can be concluded that the RK4 method gives a smaller error if compared with ode45. In practical, fiber is lossy and it is important to include higher order dispersion coefficients if the pump wavelength is close to or exactly at ZDW and when the difference between the signal and pump waves is large enough. Hence, fiber loss and higher-order dispersion coefficients were added slowly to the simulation and were solved numerically by the RK4 method. It can be concluded that fiber loss damps the peak gain of the FOPA while the bandwidth is unchanged. On top of that, the fourth-order dispersion coefficient broadens the bandwidth slightly and generates a narrow-band gain at the left and right sides of the broadband gain. However, the sixth order dispersion coefficient just moves the left and right narrow-band gains toward or away the broadband gain depending on its positive and negative signs.

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