Purification of Rotor Center's Orbit Based on Singular Value Decomposition

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Abstract
Targeting at refinement of rotor center’s orbit, a novel purification method was presented by using singular value decomposition. Firstly, the original vibration signal was reconstructed in phase space by Hankel matrix; then the attractor track matrix was decomposed with singular value decomposition. Secondly, aiming at determine reconstruction order number of singular values, the concept of energy difference spectrum of singular values was defined; then the reconstruction order number was determined according to the peak position of the energy difference spectrum. Finally, the effectiveness of the method was proved by simulation and successful purification of rotor center’s orbits coming from the miniature test-bed and the turbine generator units in the power plant. And the results of comparing the performances of the proposed method to the harmonic window and the ensemble empirical mode decomposition (EEMD) purification method show the proposed method can retain the original signal characteristic effectively and eliminate noises as much as possible. It will provide a new way for rotor center’s orbit purification.

Keywords: singular value decomposition, energy difference spectrum of singular value, rotor center’s orbit, purification

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1. Introduction
Vibration monitoring is a very important content for turbine generator units. And the rotor center's orbit can express the vibration characteristic of generator units. Different rotor center’s orbit represents different rotor condition or fault information, such as ellipse represents unbalance fault, outer eight represents misalignment, and inner eight represents oil-film whirl, et al [1]. Generally speaking, the original rotor center’s orbit can’t be used due to the noise interferences. How to eliminate noise and to obtain the clear rotor center’s orbit is the main studying content for purification.

At present, more and more studies have been done in rotor center’s orbit purification; the common used methods are digital/analog low-pass filtering and wavelet transformation. The wavelet transformation uses wavelet de-noising principle, its basic method is to decompose original signal into different frequency bands by wavelet transformation, then wave in time domain will be obtained in different frequency bands, and the purification of original signal could be realized by signal reconstruction of some frequency bands. The wavelet transformation commonly uses binary way, the amount of data and detailed signal will be lost. And the results of wavelet package transformation have the problem of energy overlapping in different frequency bands [2].

In recent years, the morphological filtering technology has become an attractive non-linear filtering method; its application has spread from power system to mechanical engineering. It has been introduced morphological filter in signal de-noising [3] and rotor center’s orbit purification [4]. Due to the selective random of structure element, its de-noising performance could not achieve the best result [5]. The ensemble empirical mode decomposition method (EEMD) [6] is a new technology in signal de-noising field, but the algorithm of EEMD has two parameters are selected artificially [7]. So these methods cannot achieve better performance by randomly selective parameters.

Singular value decomposition [8] is a common used method in signal de-noising. But there are two problems still existing in practical application, i.e., the number of array for reconstructed matrix and the effective rank order are difficult to determine [9]. For the former problem, the effective methods have been introduced [10]. For the latter one, the singular value...
entropy increment [11] and threshold method are put forward, but these methods still depend on the experiences and the de-noising result is not very good. In this paper, a novel purification algorithm is proposed based on energy difference spectrum of singular value. This method could determine the effective rank order by different singular value energy between useful signal and noises. Then the noise interferences will be eliminated successfully. And purification of rotor center’s orbit will be obtained after these procedures.

2. Principle of Singular Value Decomposition

Singular value decomposition (SVD) is a powerful numerical technique for solving systems of linear equations, and is easy to implement for order matrix calculations. It also has the advantage that it can deal with sets of equations that are close to singular, which would happen if the measured angular data were redundant because of co-linearity of vectors [12]. It is well known from linear algebra that any matrix \( A \) with \( M \) rows and \( N \) columns can be written in terms of its singular value decomposition, i.e., as the product of an \( M \times M \) column-orthogonal matrix \( U \), and an \( M \times N \) diagonal matrix \( D \), with nonnegative diagonal elements, and the transpose of an \( N \times N \) orthogonal matrix \( V \). That is,

\[
A = UDV
\]  

(1)

Here, \( D = (\text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_q), 0) \) or its transposition, which depends on \( M < N \) or \( M > N \). And 0 refers to zero matrix. Moreover, \( \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_q > 0 \), and the numbers \( \sigma_q \) are called the singular values of \( A \). If \( M = N \), \( D \) is an \( M \times M \) diagonal matrix with the singular values in decreasing order on its main diagonal. If \( M < N \), \( D \) consists of a \( M \times M \) diagonal matrix with the \( M \) singular values on its main diagonal (in decreasing order) extended on the right-hand side with a \( M \times (N-M) \) matrix of zeros. If \( M > N \), \( D \) consists of an \( N \times N \) diagonal matrix with the \( N \) singular values on its main diagonal (in decreasing order) on top of a \( (M-N) \times N \) matrix of zeros. The de-noising principle of singular value decomposition is using the energy difference between useful signal and noises, the matrix constructed by noise interrupted signal is decomposed, then the useful singular values are obtained in reconstructed procedure, and the singular values corresponding with the noise are replaced by zero [13].

Let \( x(i) \) as sample data, here \( i = 1, 2, \ldots, N \). The matrix \( A \) is constructed by phase space reconstruction theory. That is,

\[
\begin{bmatrix}
  x_1 & x_2 & \cdots & x_n \\
  x_2 & x_3 & \cdots & x_{n+1} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_n & x_{n+1} & \cdots & x_{m+n-1}
\end{bmatrix}
\]  

(2)

Here, \( 1 < n < N \), and \( m+n-1=N \), this matrix is called Hankel matrix.

The difficulty of using singular value decomposition to process the interrupted signal is how to determine the number of array for reconstructed matrix and the effective rank order. According to the conclusion from ZHAO [10] and simulation experiments, if \( N \) is even number, the column \( n = N/2 \) and the row \( m = N/2 + 1 \); While \( N \) is odd number, the column and the row may be the same as \((N+1)/2\).

In signal reconstruction procedure, determination of the effective rank order is very important. When the less singular values are selected, due to the low de-noising rank order, useful information may be lost; while the more singular values are selected, due to the high de-noising rank order, some noises will be included in reconstructed signal. Therefore, in this paper, the energy difference spectrum of singular value is defined, and the effective rank order is determined by the singular value energy between useful signal and noise interference.

WANG [13] said that signal energy could be defined by singular value as follows:
\[ E = \sum_{i=1}^{d} \sigma_i^2 \]  

(3)

Then, define the energy difference spectrum of singular value and normalized it as follows:

\[ p(i) = \frac{\sigma_i^2 - \sigma_{i+1}^2}{E} \]  

(4)

The numbers of \( p(i) \) are called energy difference spectrum of singular value. Due to energy of the useful signal is bigger than that of the noises, in the boundary of useful signal and noises, the energy difference spectrum must have a peak. So we can search the peak position as the effective rank order for the reconstructed matrix. ZHAO [9] divided signal into signal with direct-current component and signal without direct-current. When signal with direct-current component, the peak of energy difference spectrum will be gotten in the first position. If we use this peak position as the effective rank order, we will only get the direct-current component. So, in this paper, we only study the vibration signal without direct-current component [14, 15].

3. Simulation

In order to test the good purification performance of singular value decomposition, we simulate the fault vibration signal of turbine generator units with oil-film whip. And add the periodical spike pulse interference and white noises, the simulation signal showed as follows:

\[ f(x) = \sin(2\pi fx_1/f_s) + \sin(2\pi fx_2/f_s) + i(x) \]  

(5)

In which \( i(x) \) refers to the noise interferences. Take sampling frequency as \( f_s = 2000 \text{Hz} \), and \( f_1 = 25 \text{Hz} \), \( f_2 = 50 \text{Hz} \). Figure 1 shows waveform of the original simulation signal in time domain.

![Figure 1. Waveform of original simulation signal](image)

Now we use singular value decomposition method to process the signal. According to the definition of energy difference spectrum of singular value and the effective rank order of reconstructed signal will be determinted automatically by different singular value energy between useful signal and noises. Figure 2(b) shows the flow of singular value and Figure 2(c) shows the flow of energy difference spectrum. In order to observe the singular value, we select the first one hundred singular values to analyze.
In order to compare with other filtering methods, Figure 3 shows the de-noising results by mathematical morphological filter and ensemble empirical mode decomposition method.
4. Research Method

4.1. Purification of Experimental Rotor Center’s Orbit

In this section, the proposed method is applied to purify the experimental rotor center’s orbit. Figure 4 shows the rotor center’s orbit coming from the miniature rotor testbed. We use two photo-sensors to test rotor’s speed in horizon direction and vertical direction. The speed of these rotors is near 2940rpm. Let the sampling frequency equal to 512 Hz and the sample number equal to 1024. Due to the serious noise interferences, the original rotor center’s orbit is too disordered to get any fault information. Now we use the proposed method to process these signals.
Figure 4. Original rotor center's orbit of the miniature rotor testbed

Figure 5. Purified result by the propose method

Figure 5 shows the purified rotor center’s orbit by the proposed method. We can see that the purified figure clearly presents the banana shape. According to the common fault spectrum feature, we know that this figure means the fault type is misalignment fault. Figure (6) shows the purified rotor center’s orbits by harmonic wavelet window method and ensemble empirical mode decomposition method. We know that the harmonic wavelet window method also has good purified performance, but due to the intrinsic shortcoming of its algorithm, the purified result is not very satisfied. And the ensemble empirical mode decomposition method has not gotten satisfied purified result. From the comparison results we can see that the
The proposed method has good performance in purification of rotor center's orbit, it can eliminate the noise interferences effectively and reconstruct signal according to the energy difference spectrum of singular value.

Figure 6. Purification results of other two methods

4.2. Purification of Practical Rotor Center’s Orbit

In order to prove the good performance of the proposed method, in this section, it is applied to purify the practical rotor center's orbit. Figure 7 shows the practical rotor center's orbit coming from the turbine generator units in the power plant. The speed of the rotor is near 3000 rpm. Let the sampling frequency equal to 6400 Hz. Due to the serious noise interferences, the original rotor center's orbit is too disordered to get any fault information. Now we use the proposed method to process the signal.

Figure 8 shows the purified rotor center’s orbit by the proposed method. According to the common fault spectrum feature, we know that the elliptical rotor center's orbit mean
unbalanced fault for the turbine generator units. This is the good evidence to prove the effectiveness of the proposed method in purification of rotor center’s orbit.

Figure 7. Original rotor center's orbit of the turbine generator units

Figure 8. Purified result of the practical rotor center's orbit

5. Conclusions
In this paper, a novel purification method for rotor center’s orbit is presented by using singular value decomposition. The definition of energy difference spectrum of singular value is introduced and it has been used to determine the effective rank order of reconstructed matrix. When using the singular value decomposition to process the fault vibration signal, there is no
need to consider the frequency-domain feature for the sample data, and it only needs to process the signal in time-domain. Simulation and practical application show the good performance of the proposed method in rotor center’s orbit purification. It will supply a new method for fault diagnosis of rotating machinery.

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