Fault-Tolerant Control for a Class of Nonlinear Systems Based on Adaptive Observer

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Abstract
This paper addresses a fault-tolerant control scheme for a class of nonlinear systems based on the adaptive fault diagnosis method. Considering the nonlinearity of systems satisfies the Lipschitz condition, the adaptive law is designed according to the Lyapunov theory, and the proposed augmented adaptive observer can not only endure the nonlinearity but also broaden the application scopes of general adaptive observer. Then, a fault-tolerant controller is designed to compensate for the effect of the faults and guarantees the closed-loop system stable. Finally, a numeral example is presented to illustrate the effectiveness and feasibility of the addressed approach.

Keywords: Nonlinear system, Fault detection and diagnosis, Fault-tolerant control, Adaptive observer, Adaptive law

1. Introduction
A fault is defined as an unexpected change in a system with component malfunction or variation in operating condition. Faults in a dynamic system can take many forms. They can be actuator faults, sensor faults, unexpected abrupt changes of some parameters and so on. These faults may result in unsatisfactory performance or instability. So, it is important that faults can be promptly detected and appropriate remedies can be applied. The safety, reliability and maintainability in actual systems and industrial process have motivated researchers to concentrate on the so-called fault-tolerant control (FTC) [1-3]. FTC is primarily meant to ensure safety, i.e., the stability of a system after the occurrence of a fault in the system. There are two approaches to synthesize controllers that are tolerant to system faults. One approach, known as passive FTC, aims at designing a controller which is a prior robust to some given expected faults. Another approach, known as active FTC, relies on the availability of a fault detection and diagnosis (FDD) block that gives, in real-time, information about the nature and intensity of the fault. This information is then used by a control reconfiguration block to adjust online the control effort in such a way to maintain stability and to optimize the performance of the faulty systems. Researches on FDD for systems have long been recognized as one of the important aspects in seeking effective solutions to an improved reliability of practical control systems. Accurate fault estimation can determine the size, location and dynamic behavior of the fault, which automatically indicates FDD, and has thus attracted interests recently. Many methods have been proposed for FDD, e.g., parity space-based approach [4], Kalman filters approach [5], parameter identification-based approach [3] as well as observer-based approach [7-11]. Observer-based FDD, including sliding model observer-based FDD [10, 11] and adaptive observer-based FDD [12-15], is one of the most effective methods and has obtained much more attention. So far, various observer-based FDD approaches have been reported in the literatures.

People have took more and more attention to the method of adaptive observer and obtained many valuable results [12-15]. Generally, the adaptive observer can only handle the constant fault of system. For example, the design of adaptive observer based on fault detection and diagnosis by estimating parameter for the Lipschitz nonlinear system is presented in Ref. [12]. For a class of nonlinear systems with unknown parameters, Ref. [13] addresses the approach of constructing adaptive observer through Lyapunov theory. Ref. [14] considers the problem of fault diagnosis via a robust adaptive observer for nonlinear system, the designed observer can deal with well the uncertainty of system. However, the fault of above mentioned were all supposed to be a constant. For time-varying value fault, Ref. [15] presents the
approach of design augmented adaptive observer and broadens the scopes of observer application. A recursive algorithm for joint estimation of the state vector and the parameter vector related to faults was developed by Xu and Zhang based on high gain adaptive observers [16]. Overall, the basic idea behind the use of the observer for FDD is to estimate the state or output of the system from the measurement by using some type of observers, and then to construct a residual by a properly weighted state or output error. The residual is then examined for the likelihood of faults by using a fixed or adaptive threshold. However, it is difficult to design the fault-tolerant controller to the faulty system.

This paper designs the fault tolerant controller for nonlinear systems on the basis of adaptive fault diagnosis observer. Considered the nonlinearity of fault and systems and the time-varying value fault, the augmented adaptive law is designed based on the Lyapunov theory. The designed augmented adaptive observer can not only endure the nonlinearity, but also broaden the application scopes of general adaptive observer. Then, based on the obtained fault information and the nonlinearity of systems, the fault-tolerant controller is proposed to ensure the fault system is stable.

In the present paper, the notations are rather standard. $$\mathbb{R}^n$$ denotes the set of real number and complex plane respectively; $$\mathbb{R}^{n \times n}$$ is the set of all $$n \times n$$ real matrices; $$I$$ is the identity matrix with appropriate dimensions; The superscript $$T$$ stands for a matrix transposition; $$A^T$$ denotes the generalized inverse of matrix $$A$$; $$\lambda_{\min}(P)$$ and $$\lambda_{\max}(P)$$ refer to the minimal and maximal eigenvalues of the matrix $$P$$ respectively; $$P > 0$$ (or $$P < 0$$) indicates the symmetric matrix $$P$$ is positive (or negative) definite; $$\forall$$ means “for all”; The vector norm $$\|x\|$$ is defined as $$\|x\| = \sqrt{x^T x}$$.

For a symmetric matrix, $$^*$$ denotes the matrix entries implied by symmetry.

The paper is organized as follows. Section 2 contains a description of FTC problem formulation for a class of nonlinear systems. In Section 3, a robust and full-order observer design is discussed, and then the actuator fault detection problems are considered. In Section 4, we develop a kind of adaptive fault diagnosis observer and based on it an actuator fault tolerant controller is put forward in section 5. The simulation is given in Section 6. In Section 7, some conclusions for the full paper are drawn [19, 20].

2. Problem Formulation

The following nonlinear system with actuator fault considered in the paper can be described as

$$\dot{x}(t) = Ax(t) + Bu(t) + g(t,x) + Df(t)\phi(x,y,u)$$
$$y(t) = Cx(t)$$  \hspace{1cm} (1)

where $$x(t) \in \mathbb{R}^n$$, $$u(t) \in \mathbb{R}^p$$, $$y(t) \in \mathbb{R}^q$$ is the state vector, input vector and measurable output vector respectively; $$A, B, C$$ and $$D$$ are known matrices with appropriate dimensions; the function $$g(t,x)$$ stands for the nonlinear perturbation; $$f(t)$$ is the unknown time-varying parameter vector and assumed to be bounded when fault occurs, that is, there exists a positive constant $$f_0$$ such that $$\|f(t)\| \leq f_0$$; When no faults occur, $$f(t) = 0$$, $$\forall t \geq 0$$, where $$f_0$$ is a known constant; $$\phi(x,y,u)$$ is the nonlinear function of describing the property of system fault. The functions $$g(t,x)$$ and $$\phi(x,y,u)$$ are real nonlinear vector functions satisfying the Lipschitz constraint [17].

The system (1) is assumed to be asymptotically stable since no feedback control exists.

Now, one can give the following assumptions which are necessary for the rest of the paper.

Assumption 1

$$\text{rank} \begin{bmatrix} sI - A \\ C \end{bmatrix} = n, \quad \forall s \in \mathbb{C}^+ = \{ s \in \mathbb{C} : \text{Re}(s) \geq 0 \}$$

Assumption 1 implies the system (1) is observable.
Assumption 2

The functions \( g(t,x) \) and \( \phi(x,y,u) \) are real nonlinear vector functions satisfying the following

Lipschitz constraint:

\[
\|g(t,x_1) - g(t,x_2)\| \leq \|U_x(x_1 - x_2)\|, \quad \forall (t,x_1),(t,x_2) \in \mathbb{R} \times \mathbb{R}^n \tag{2a}
\]

\[
\|\phi(x_1,y,u) - \phi(x_2,y,u)\| \leq \|U_y(x_1 - x_2)\|, \quad \forall (x_1,y,u),(x_2,y,u) \in \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}^n \tag{2b}
\]

where \( U_x, U_y \in \mathbb{R}^{n \times m} \), \( U_x \) and \( U_y \) are a known constant matrix. Provided that \( g(t,x_1) = g(t,x_2) \), there exist matrices \( \delta_x, \delta_y \) such that

\[
\|g(t,x_1) - g(t,x_2)\| \leq \delta_x \|x_1 - x_2\|
\]

Likely, substitution of \( U_x = F_x C \) into inequality (2a) yields

\[
\|g(t,x_1) - g(t,x_2)\| \leq \|F_x C(x_1 - x_2)\| \leq \|C(x_1 - x_2)\| \leq \delta_x \|x_1 - x_2\|
\]

Assumption 3 [18]

For a positive definite matrix \( P > 0 \), there exists a matrix \( L \) with appropriate dimensions such that the following equations

\[
(A - LC)^T Q_1 + Q_1 (A - LC) = -P 
\]

\[
C^T Z = Q D 
\]

\[
H^T Q_1 = Z C, \quad \text{where} \quad H = [D \ 0 \ I] 
\]

have a positive definite matrix solution \( Q_1 > 0 \), where \( H, Z \) are known constant matrices.

\[
(A - LC)^T Q_1 + Q_1 (A - LC) = -P 
\]

\[
C^T Z = Q D 
\]

It can be concluded that the system discussed in this paper is controllable from the assumption 1. Moreover, the equation (3) in assumption 3 is the famous Lyapunov equation and equation (4) in assumption 3 is called matching condition. The Lyapunov equation with a matching condition has been used in many control design discussions, especially in adaptive control and adaptive and robust observer design problems. The Kalman–Yakubovich–Popovlemma [18] shows that if a matrix \( L \) can be chosen such that the transfer function matrix \( G(s) = C(sI - (A - LC))^T H \) with \( H = [D \ 0 \ I] \) is strictly positive real (SPR), then for a positive definite matrix \( Q_1 \), there exist a positive definite matrix \( P \) and a matrix \( Z \) to satisfy equations (3)(4). The matching condition seems to be restrictive theoretically, but fortunately, for many practical control systems, particularly mechanical systems. Moreover, the LIM tools of MATLAB provide a way to find the solutions for condition (3)(4) when there really exist solutions.
by using the known input $u(t)$ and the measurement output $y(t)$. The second goal is to work out an efficient fault tolerant control scheme by using the estimated states and faults.

Before the fault detection and diagnosis observers design, one presents the famous Barbalat’s lemma.

**Lemma 1 (Barbalat’s Lemma) [18]** If $\phi : [0, \infty) \to \mathbb{R}^n$ is a uniformly positive function for $t \geq 0$, and if the limit of the integral $\lim_{t \to \infty} \int_0^t \phi(\tau) d\tau$ exists and is finite, then $\lim_{t \to \infty} \phi(t) = 0$.

### 3. Full-Order Fault Detection Observer Design

Based on the system model given by (1), the full-order fault detection observer is chosen as follows:

$$
\dot{x}(t) = Ax(t) + Bu(t) + g(t,x) + Df_\delta(\tilde{x}, y, u) + L(y(t) - \tilde{y}(t))
$$

$$
\tilde{y}(t) = C\hat{x}(t)
$$

(5)

where $\tilde{x}(t) \in \mathbb{R}^n$ is the estimation vector of system state, $\tilde{y}(t) \in \mathbb{R}^p$ is the estimation vector of output. From the assumption 1 and assumption 3, one can know that it is solvable to choose appropriate matrix $L$ of the observer to guarantee matrix $(A - LC)$ is stable.

Next defining the error of the state estimation and the output estimation as $w(t)$ and $\nu(t)$ respectively as follows

$$
w(t) = \tilde{x}(t) - x(t)
$$

$$
\nu(t) = \tilde{y}(t) - y(t)
$$

(6)

so, the following error dynamic system between system (1)(5) is

$$
\dot{w}(t) = (A - LC)w(t) + g(t,\tilde{x}) - g(t,x) + D[f_\delta(\tilde{x}, y, u) - f(\tilde{x}, y, u)]
$$

$$
\nu(t) = Cw(t)
$$

(7)

One has the following result.

**Theorem 1** For the system (1), the observer (5) and the error equations (7), if there exist scalars $\delta_0 > 0$, $\delta_\alpha > 0$ in assumption 2, and matrices $Q_0 > 0$, $P > 0$ and $\alpha_\delta = \lambda_{\text{max}}(P) - 2\|f_\delta \|_\lambda \delta_\alpha > 0$ in assumption 3, thus the state estimation $\hat{x}(t)$ from the robust full-order observer determined by (5) converges to the actual state $x$ asymptotically when no actuator fault occurs, that is $\lim_{t \to \infty} w(t) = 0$, moreover, $\lim_{t \to \infty} \nu(t) = 0$.

**Proof:** Under assumption 3, there exist matrices $Q_1 > 0$, $P > 0$ such that equalities (3) (4) hold. Next, defining a Lyapunov function

$$
V(t) = w^T(t)Q_1 w(t)
$$

(8)

the derivative of $V(t)$ along with the error equations (7)

$$
\dot{V}(t) = w(t)^T \left[ Q_1 (A - LC) + (A - LC)^T Q_1 \right] w(t)
$$

$$
+ 2w(t)^T Q_1 Df_\delta \left[ \phi(\tilde{x}, y, u) - \phi(x, y, u) \right] + Q_1 \left[ g(t,\tilde{x}) - g(t,x) \right].
$$

Applying assumption 2 and assumption 3 to the above inequality, one can have

$$
\dot{V}(t) \leq -w^T(t)Pw(t) + 2Cw(t)Zf_\delta \| \tilde{x} - x \| 
$$

$$
+ Q_1 \| \tilde{x} - x \| - w^T(t)Pw(t) + (2\| \nu(t) \|Zf_\delta + Q_1) w(t).
$$

Letting $\alpha_\delta = \lambda_{\text{max}}(P) - 2\|f_\delta \|_\lambda \delta_\alpha$, it is derived
\[
\dot{V}(t) \leq -\alpha \|w(t)\|^2 \leq 0 \quad (8b)
\]

Based on Lyapunov stability theory, \( \dot{V}(t) \leq -\alpha \|w(t)\|^2 \leq 0 \) means that the equilibrium points \( w(t) = 0 \) of error dynamic system (7) is stable. Now integrating equation (8b) from zero to \( t \) yields \( V(t) + \int_0^t \alpha \|w(\sigma)\|^2 d\sigma \leq V(0) \) which implies that \( 0 < \int_0^t \alpha \|w(\sigma)\|^2 d\sigma \leq V(0) \). Since \( V > 0 \) and \( w(t) > 0 \), as \( t \) approaches infinite, the above integral is always less than or equal to \( V(0) \), so the limit of the integral \( \lim_{t \to \infty} \int_0^t \alpha \|w(\sigma)\|^2 d\sigma \) exists and is finite. So by Barbalat’s lemma, one can obtain \( \lim_{t \to \infty} w(t) = 0 \), furthermore, the observer (5) is asymptotically convergent, that is, the state estimation \( \hat{x}(t) \) from the robust full-order observer determined by (5) converges to the actual state \( x(t) \) asymptotically. The proof is completed.

When actuator fault occurs, it is actually a kind of disturbance and there exists a control law to robustly control it, so it will affect the output. In other words, the output error \( \nu(t) \) is affected by the fault \( f(t) \), unaffected by disturbance \( \phi(x,t) \) and asymptotically approaches to zero whenever \( f(t) = 0 \). For this reason, the robust full-order fault detection observer determined by (5) can serve as fault detection observer. Based on this, the occurrence of fault can be detected by the following decision rule:

\[
\|\nu(t)\| \leq \epsilon \quad \text{no fault occurs} \quad (9a)
\]

\[
\|\nu(t)\| \geq \epsilon \quad \text{at least one fault has occurred} \quad (9b)
\]

Where \( \epsilon \) is a threshold which is set artificially for fault detection, one can determine whether the system is affected by some faults or not.

Suppose that there are \( N \) types of faults on system (1), the set of these faults can be devoted for \( F = \{\phi(x,y,u), \phi(x,y,u) \cdots \phi(x,y,u)\} \). For \( \phi(x,y,u), i = 1,2 \cdots N \), its fault parameter \( f(t) \) is bound, that is, \( \|f(t)\| \leq f_b \) where \( f_b \) is a known constant.

4. Adaptive Fault Diagnosis Observer Design

In this section, an adaptive fault diagnosis observer will be proposed for FDD, which can provide the information of the states and faults. The information is sent to the controller to obtain the control law which is sent to the actuator.

In order to estimate better the states and faults, one designs the adaptive fault diagnosis observer as follows.

\[
\dot{\hat{w}}(t) = (A - LC)\hat{w}(t) + Bu(t) + g(t,\hat{w}) - \Gamma(t)Z(y(t) - \hat{y}(t)) + Df(t)\hat{\phi}(\hat{w},y,u)
\]

\[
\dot{\hat{y}}(t) = C\hat{w}(t) \quad (10)
\]

where \( \hat{w}(t) \in \mathbb{R}^n \), \( \hat{y}(t) \in \mathbb{R}^p \) are the state vector and output vector of the fault diagnosis observer respectively. \( \Gamma(t) \) is the adaptive law to be designed. Assumed that \( f(t) = f_b \) and \( f_b \leq f_b \) before detecting the faults, and \( f(t) = f_0 \) where \( f_0 \) is a known constant after having detected the faults. The purpose of this section is to design the adaptive law \( \Gamma(t) \) such that

\[
\lim_{t \to \infty} \hat{w}(t) = 0
\]

\[
\lim_{t \to \infty} (f(t) - \hat{f}(t)) = \lim_{t \to \infty} \hat{f}(t) = 0 \quad (11)
\]
Remark 1 It should be pointed out that $\hat{f}(t) = f - \tilde{f}(t)$ from the equations which can guarantee that the adaptive law is suitable to estimate the fault, and the error is equal to zero when the fault is detected, where $f$ stands for the fault value of system, $\hat{f}(t)$ is the estimation of the fault, $\tilde{f}(t)$ is the error between the real value and the estimation of the fault.

According to the observer equation (10), the error dynamic equation can be characterized as

$$
\dot{\hat{w}}(t) = (A - LC)\hat{w}(t) + g(t, w) - g(t, \hat{w}) - \Gamma(t)Z(y(t) - \hat{y}(t)) + Df(t)\phi(\tilde{\tilde{w}}, y, u) \tag{12}
$$

\[
\hat{y}(t) = C\hat{w}(t)
\]

One present the following result.

**Theorem 2** For the system (12) and adaptive fault diagnosis observer (10), the adaptive law $\Gamma(t)$ is determined by

$$
\Gamma(t)(\kappa) = \eta\|Z(y(t) - C\tilde{x}(t))\| \tag{13}
$$

thus $\lim_{t \to \infty} \hat{w}(t) = 0$ and $\lim_{t \to \infty} \hat{f}(t) = 0$

**Proof:** Consider the following Lyapunov function candidate

$$
V(t) = \tilde{w}^T(t)Q_w(t) + \eta^{-1}\tilde{f}^2(t) \tag{14}
$$

where $Q_w > 0$ is defined in assumption 3. The derivative of $V(t)$ along with the error dynamic system (12) is

$$
\dot{V}(t) \leq \tilde{w}^T(t)\left[\begin{array}{c}
\tilde{w}^T\left(\frac{Q}{A - LC} + \frac{Q}{A - LC}\right)\tilde{w} + 2\tilde{w}Q\gamma \left[\begin{array}{c}
\Theta(t) - \tilde{y}(t) + \tilde{y}(t) - \tilde{y}(t) - \tilde{y}(t) - \tilde{y}(t)
\end{array}\right]
\end{array}\right] + \eta^{-1}\tilde{f}(t)\dot{\tilde{f}}(t)
$$

where $\alpha_2$ is an unknown parameter and is decided by (17), the above inequality is also equivalent to

$$
\dot{V}(t) \leq \left(-Q_1 + \alpha_2 - 2\delta_1\right)\tilde{w}(t) + \left(\tilde{f}(t) - f(t)\right)\tilde{w}(t) + \eta^{-1}\tilde{f}(t)\dot{\tilde{f}}(t) \tag{15}
$$

Supposing $\Gamma(t) = \eta > 0$, $\Gamma(t)(\kappa) = \eta\|Z(y(t) - C\tilde{x}(t))\|$ and combining with (11), one can have

$$
(\tilde{f}(t) - f(t))\tilde{w}(t) + \eta^{-1}\tilde{f}(t)\dot{\tilde{f}}(t)
$$

Inequality (15) can be recast to the following inequality

$$
\dot{V}(t) \leq \left(-Q_1 + \alpha_2 - 2\delta_1\right)\tilde{w}(t)^2 \tag{16}
$$

Letting

$$
\alpha_2 \leq \lambda_{\text{min}}(Q_1) + 2\delta_1 \tag{17}
$$
Furthermore, it is obtained
\[ \dot{V}(t) \leq -\varphi(t) < 0 \] (18)

where \[ \varphi(t) = (Q_1 - \alpha_2 + 2\delta_2) \| \tilde{w}(t) \|^2 . \]

By Lyapunov stability theory, it is clear that the error dynamic system (12) is stable. Inequality (18) can be obtained equivalently
\[ \dot{V}(t) + \varphi(t) < 0 \] (19)

Integrating inequality (19) from 0 to \( t \), one has \( V(t) + \int_0^t \varphi(\tau) \, d\tau \leq V(0) \), that is \( 0 < \int_0^t \varphi(\tau) \, d\tau \leq V(0) \). It is known that \( V(0) > 0 \) and \( \varphi(t) > 0 \), so \( \int_0^t \varphi(\tau) \, d\tau \) exists and is finite. According to lemma 1, it is easy to know that \( \lim_{t \to \infty} \varphi(t) = 0 \) which implies \( \lim_{t \to \infty} \tilde{w}(t) = 0 \), furthermore \( \lim_{t \to \infty} \tilde{f}(t) = 0 \). The proof of theorem 2 is completed. \( \Box \)

Notice that theorem 2 implies the design adaptive observer can estimate well the state and the fault under the adaptive law (13).

5. Fault-Tolerant Controller Design Based On

In this section, the estimates of states and faults provided in section 4 are used to design the fault-tolerant controller to ensure the faulty system is stable. Now, one will presents some assumptions to restrict the nonlinearity.

**Assumption 4**

If there exists a positive definite symmetry matrix \( Q > 0 \), continuous and local bounded function \( \mu(x,t) \) such that the following statements hold

1. \( A - BB^TQ \) is stable
2. \( \| x(t)Qg(t,x) \| \leq \mu(x,t) \| x(t)QB \| . \)

then the nonlinear term \( g(t,x) \) of system (1) is called matchable and bounded.

**Assumption 5**

The matrix pair \((A,B)\) is controllable and \( \text{rank}([B\ D]) = \text{rank}(B) \).

Under assumption 4-5, the FTC controller can be designed to be in the forms of
\[ u(t) = u_1(t) + u_2(t) \] (20)

Where
\[ u_1(t) = -B^TQ\tilde{w}(t) - B_iD\tilde{f}(t) \] (21)
\[ u_2(t) = g(t,\tilde{w})\mu(t,\tilde{w})B^TH\tilde{w}(t) \]
\[ \left\| \mu(t,\tilde{w})B^TH\tilde{w}(t) \right\| + \frac{\xi}{2} \] (22)

\( \xi \) is a sufficiently small scalar, matrix \( B_i \) is chosen to guarantee \( (I - BB_i)D = 0 \), matrix \( Q > 0 \), the function \( \mu(x,t) \) is known in assumption 4, matrix \( H \) or \( Z \) is determined by assumption 3.

From the style of the controller (20), one can know that the controller \( u_1(t) \) is constructed by the estimations of the states and the faults, and the controller \( u_2(t) \) is designed based on the nonlinearity term \( g(x,t) \) of the system (1). The main results are summarized in the following
Theorem 3: Under assumption 1-5, there exist the fault-tolerant controllers (20)-(22) such that the system (1) is asymptotically stable when the faults occur or not.

Proof: Applying the controllers (20)-(22) to the system (1), the closed-loop dynamics system can be written as

$$\dot{x}(t) = (A - BB^T Q)x(t) + BB^T Q\hat{w}(t) + (I - BB)Du(t) + BB_Df(t) + Bu_z(t) + g(t, x)$$  \hspace{1cm} (23)

from assumption 5

$$\dot{x}(t) = (A - BB^T Q)x(t) + BB^T Q\hat{w}(t) + Du(t) + Bu_z(t) + g(t, x)$$  \hspace{1cm} (24)

Consider a Lyapunov function

$$V(t) = \bar{w}^T(t)Q\bar{w}(t) + g^T(t)Q\bar{w}(t) + \rho x^T(t)Qx(t)$$  \hspace{1cm} (25)

where $\rho > 0$ is a positive scalar and $Q_z > 0$ is a positive define matrix. The positive define matrices $Q_i > 0$ and $Q > 0$ are known and defined in assumption 3 and assumption 4 respectively.

By equality (22) and equality (25), one can derive

$$\rho \bar{w}^T(t)Q[Bu_z(t) + g(t, \bar{w})] \leq \rho \left[ \bar{w}^T(t)Qg(t, \bar{w}) + \bar{w}^T(t)QBu_z(t) \right] \leq \frac{1}{2} \rho \xi$$  \hspace{1cm} (26)

So,

$$\rho x^T(t)Q[Bu_z(t) + g(t, \bar{w})] \leq \frac{1}{2} \rho \xi + \varpi$$  \hspace{1cm} (27)

where $\varpi$ is the bounded norm of the translation error between the state $x$ and the estimation $\bar{w}$.

Considering (25) and (27), it is yielded

$$\dot{V}(x) = \epsilon_1 \|x(t)\|^2 + \epsilon_2 \|x(t)\|^2 + \epsilon_3 \|x(t)\|^2 + \frac{1}{2} \rho \xi + \varpi$$  \hspace{1cm} (28)

and

$$-\epsilon_1 = \lambda_{max} \left( \rho U - \frac{\rho QBB^T}{\beta_1} + \frac{DD^T}{\beta_2} \right)$$

$$-\epsilon_2 = \lambda_{max} \left( Q - \rho QBB^T \right)$$

$$-\epsilon_3 = 2\beta_3 - \frac{1}{2} \beta_2$$  \hspace{1cm} (29)

where $\beta_i, (i = 1, 2, 3)$ are constants.

Choosing appropriate scalar $\rho$ such that $\epsilon_i, (i = 1, 2, 3) > 0$ holds.

thus, when

$$\left( \epsilon_1 \|x(t)\|^2 + \epsilon_2 \|x(t)\|^2 + \epsilon_3 \|x(t)\|^2 \right) \geq \frac{1}{2} \rho \xi + \varpi$$

$$\dot{V}(x) < 0$$ also holds.

Based on the Lyapunov theory, the system (1) with actuator faults under the controller
(20)-(22) is still asymptotically stable. This ends the proof.

Now we can begin to discuss the algorithm of the adaptive observer and the fault-tolerant controller as follows.

Input: the system (1) and matrix $Z$, Lipschitz constants $\delta_x$ and $\delta_y$.

Output: controller parameters $B_i$ in equation (21), $\xi$ in equation (22)

**Step 1:** Choosing the matrix $L$ to ensure that $A - LC$ is stable and the transfer function matrix $C(sI - (A - LC)^{-1}B$ is restrict positive real.

**Step 2:** Choosing a positive definite matrix $P > 0$ such that $\alpha_i = \lambda_{\text{max}}(P) - 2\|Z\|f_0\delta_x - \lambda_{\text{max}}(P)\delta_y > 0$, then solving equations (3)(4) in $Q_i$. If there is no feasible solution $Q_i$ to equations (3)(4), thus the step 2 will be repeated and another expected matrix $P > 0$ is choose until there is a feasible solution $Q_i$ to equations (3)(4).

**Step 3:** By theorem 2, compute out the adaptive law $\eta$ of the observer.

**Step 4:** Under assumption 4, selecting a matrix solution $P > 0$ such that $(A - BB^TQ$ is stable.

Choosing a matrix $B_i$ such that $(I - BB_i)D = 0$.

**Step 5** Compute the parameter $\xi$ of controller $u_x(t)$ in theorem 3.

By step 1 – step 5, we have described the approach of designing the adaptive observer and the fault-tolerant controller.

**6. Simulation**

Consider the nonlinear system (1) with the following parameters:

$$
\begin{align*}
A &= \begin{bmatrix} 5.2 & 2.4 \\ 4.7 & 8.6 \end{bmatrix}, \\
B &= \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \\
C &= \begin{bmatrix} 4 & 6 \\ 9 & 1 \end{bmatrix}, \\
W &= \begin{bmatrix} 1.52 & 6.51 \\ 0.83 & 7.49 \end{bmatrix}, \\
W &= \begin{bmatrix} 1.6 & 1.8 \\ 3.2 & 4.2 \end{bmatrix}, \\
g(t,x) &= \begin{bmatrix} 0.8\sin(x_1 + 2x_2) \\ -0.2\cos(x_1 + 2) \end{bmatrix}, \\
\mu(x,y,u) &= 0.6\sin(x + 2y + u)
\end{align*}
$$

It is easy to prove that matrices $A$ and $C$ satisfy assumption 1, matrices $A,B,D$ satisfy assumption 5.

The Lipschitz constants in assumption 2 can be given as $\delta_x = 1.58$ and $\delta_y = 2.69$. The bound of the faults can be presented as $f_0 = 0.2$, $f_y = 0.2$. The matrix $Z$ in assumption 2 and $\mu(t,x)$ in assumption 4 are $Z = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.9 \end{bmatrix}$ and $\mu(t,x) = \sin(2x + t)$ respectively.

So, we can design the fault detection observer and fault diagnosis observer. Using the above data, we can get the following simulation results.

First, choosing matrix $L = \begin{bmatrix} 0.05 & 0.09 \\ 0.04 & 0.16 \end{bmatrix}$ to make $(A - LC$ is stable. and solving the matrix equation of assumption 3 by Matlab software, we have

$$
\begin{align*}
P &= \begin{bmatrix} 2.0163 & 1.9145 \\ 0.6986 & 3.2569 \end{bmatrix}, \\
Q_i &= \begin{bmatrix} 0.3497 & 0.2213 \\ 0.2213 & 0.4591 \end{bmatrix}
\end{align*}
$$

According to the result, we can further obtain that scalar $\alpha_i = 0.0259$ of theorem 1.

Supposing $\alpha_2 = 2.36$, the adaptive law $\Gamma(t)$ of observer can be designed as $\eta = 0.3874$.

Next, we will give the design of the fault-tolerant controllers. Considering the condition of assumption 4, assumed the nonlinear function $\mu(t,x) = \sin(2x + t)$, we can obtain the positive definite symmetry matrix

$$
\bar{Q} = \begin{bmatrix} 4.3652 & 0.2463 \\ 0.2463 & 1.2456 \end{bmatrix}
$$

Solving $(I - BB_i)D = 0$ in $B_i$, we have

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so, the controller $u_1(t)$ can be derived from equation (21).

Considering the matrix $H = \begin{bmatrix} 0.68 & 0.26 \\ 0.22 & 0.96 \end{bmatrix}$, we have $Q_2 = \begin{bmatrix} 0.6325 & 0.1058 \\ 0.1058 & 0.7754 \end{bmatrix}$.

Supposing $\omega = 0.25$, $\rho = 0.02$, $\beta_1 = 0.8$ and $\beta_2 = 0.5$, using equation (30), we can get $\xi_1 = 0.2$, $\xi_2 = 0.14$, $\xi_3 = 0.7$. Hence, the parameter $\xi$ of controller $u_1(t)$ can be calculated as $\xi = 2.3084$, the controller $u_2(t)$ can also be obtained from equation (22). Therefore, the design of the fault tolerant controller $u(t) = u_1(t) + u_2(t)$ for the system is completed. The example describes the procedure of designing fault tolerant controllers for a class of nonlinear systems with actuator faults.

7. Conclusion

In this paper, we consider the fault detection and diagnosis for Lipschitz nonlinear system with time-varying fault based on adaptive fault observer. The adaptive law developed can not only guarantee the nonlinearity of system, but also broaden the application scopes of general adaptive observer. Then, we design the fault-tolerant controller on the basis of information obtained by the observer. At last, the simulation shows that the method proposed in this paper is easily to solve and suitable to operate in the computer.

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References


