Application of Uncorrelated Learning from Low-Rank Dictionary in Blind Source Separation

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Abstract

This paper proposes a kind of method about signal BOA estimation from the aspect of sparse decomposition. The whole interested space is divided into several potential angles of arrival to establish an over-complete directory to convert the estimation problem of signal DOA to sparse representation problem. A MMV array is formed by data received from multiple snapshots, then using optimization method of joint sparse constraint to solve the problem. First, make singular value decomposition on received data array to connect the each snapshot data, then using the sparse representation problem of \( l_0 \) bounded to solve the problem. To improve the anti-noise performance of algorithm, the paper applies similar Sigmoid function of two parameters to approximate \( l_0 \) norm. This method applies to the DOA estimation of narrow-band and broad-band signal. \( JSL_0 - SVD \) shall be used for solving MMV problem, which achieves joint sparse constraint of all frequency of reception matrix of broadband signal, to make array elements spacing break through the limitation of half wavelength and improve resolution of DOA signal.

Keywords: Blind source; Source separation; Joint sparse; Smooth norm

1. Introduction

Estimation problem of DOA has new solving method with the sparse representation [1-4] and compressed sensing [5-8] methods appear recently. The estimation of DOA is expressed as the sparse reconstruction problem of beam space by Fuchs [9]; MMV (multiple measurement vectors) and matching pursuit are combined by Cotter [10] to solve the joint sparse reconstruction problem in DOA estimation; singular value decomposition (SVD) of subspace method and sparse reconstruction method based on \( l_1-SVD \) are combined by Malionlov [11] to achieve the estimation of DOA. When the number of signal source is unknown, the performance of \( l_1-SVD \) algorithm may decline significantly; the calculation of this method is very large and is affected by noise easily. This paper makes the estimation problem of remote field signal transformed into a joint sparse represented problem. The angle space is divided into \( N \) parts from \( \theta_1 \) to \( \theta_N \); it is assumed that there is signal DOA possibly on each aspect, therefore the distinguishable degree of angle is \( N \). Number of actual space signal source is \( K \); the number of incident angle is \( K \) of \( N \); \( N \gg K \) in general, so it can be defined that the signal source vector is \( K \) sparse and there is signal source on corresponding nonzero value position, and the zero value presents no signal source; the obtained signal source vector is used to search the nonzero value to get direction estimation of each signal sources. Received array is a uniform linear array (ULA). The paper adopts a kind of extended smooth \( l_0 \) approximation algorithm—joint smooth \( l_0-SVD(JSL_0 - SVD) \) to achieve the estimation of signal DOA. First, making singular value decomposition on received on data array to reduce the dimension of measured data and making estimation of signal DOA expressed as joint sparse representing problem of signal subspace at the same time; then adopting joint smooth \( l_0 \) norm approximation algorithm to achieve the estimation of signal DOA. To improve anti-noise capability of algorithm, this paper adopts similar Sigmoid function of two parameters to approximate \( l_0 \) norm. At the same time, this paper extends \( JSL_0 - SVD \) to solve occurred MMV problem in estimation of board signal DOA. \( JSL_0 - SVD \) shall be used for solving MMV problem, which achieves joint sparse constraint of all frequency of reception matrix of broadband signal, to make array elements spacing break through the limitation of half wavelength and improve resolution of DOA signal. In addition, \( JSL_0 - SVD \) also includes the following advantages: better anti-noise capacity; higher computational efficiency; applied to coherent and incoherent signal.
2. Model Construction

2.1. Signal Model

The object of estimation of signal DOA is to find the angle of arrival incident on signal source of array. Known information includes geometric construction of array, parameter of medium of signal transmission, received data of array element. This section firstly considers the DOA estimation of narrow-band signal and DOA estimation of broadband signal will be discussed in next section.

![Figure 1. ULA receiving array](image)

Considering ULA array shown in Figure 1. \( M \) of each isotropic receiving array receives \( K \) of remote field static incident signal. The most left array element is reference array element, distance of array element is \( d = \lambda / 2 \), \( \lambda \) is the wave length. The angle of incident of is \( \theta_k \).

Received signal of array element can be expressed as the following linear equation:

\[
y(t) = A(\theta)s(t) + w(t)
\]

In which, \( y(t) = [y_1(t), y_2(t), \ldots, y_M(t)]^T \) is received signal vector, \( w(t) \) unknown noise vector and \( s(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T \) is vector of signal source. \( A(\theta) = [a(\theta_1), a(\theta_2), \ldots, a(\theta_K)] \) is oriented matrix, in which, \( a(\theta_i) \) is vector of length \( M \) and \( a(\theta_m) = e^{j(\pi m \sin \theta)} \), \( m = 1, 2, \ldots, M \).

Estimation problem of signal DOA is the direction of angle \( \theta = \{\theta_k\} \) of signal source that estimated by received signal vector \( y(t) \).

In spite of DOA estimation based on single snapshot data has its application values, DOA estimation with multi-snapshot data may occur in actual application frequently. Considering the time sampling is formula (1), narrow-band signal DOA estimation problem with multi-snapshot data may be expressed as:

\[
Y = A(\theta)S + W
\]

In which, \( Y = [Y(t_1), Y(t_2), \ldots, Y(t_J)] \), \( J \) is snapshot number. The definition of \( S \) and \( W \) is the same as \( Y \).

2.2. DOA Estimation Expressed on the Basis of Sparse Directory

To transform DOA estimation into a sparse representation problem, all possible over-complete angle of arrival shall be introduced to express \( A \). Firstly, dividing the whole interested space into several potential angle of arrival, such as \( \Omega = \{\theta_1, \theta_2, \ldots, \theta_N\} \), \( N \neq K \) in general; then
all potential angle of arrival can be used for constructing a over-complete oriented matrix
\[ \Phi = \begin{bmatrix} a(\theta_1) & a(\theta_2) & \ldots & a(\theta_N) \end{bmatrix} \]. \( \Phi \) is known and not relevant to DOA of actual signal source.

\( N \times 1 \) vector quantity \( S(t_j) \) shall be expressed as location of signal sources; and when \( \vec{a}_n = \theta_k \), \( n \) element of \( S(t_j) \) is non 0, or it is 0. DOA information signal source can be obtained from the location of non 0 value of \( S(t_j) \). Of course, DOA of actual signal may not be equal to certain \( \vec{a}_n \) all the time. However, if \( \Omega \) is enough intensive, thus certain \( \vec{a}_n \) exists to make \( \vec{a}_n \approx \theta_k \); existing deviation can be expressed approximately as noise. Analyzing from the above, the DOA estimation of signal can be expressed as: \( Y = \Phi S + W \)

For only considering the situation of static signal in this paper, DOA of signal is a time-invariant vector in the whole measuring progress; each line of non 0 value in matrix \( S \) is appeared in the same line. In other words, only \( K \) behavior is non 0 in matrix \( S \). Therefore, DOA estimation of signal can be expressed as MMV problem of joint sparse matrix \( S \) is found by observation data \( Y \).

3. DOA Estimation of Narrow-Band Signal

3.1. Multiple Parameter \( l_0 \) – norm Approximation

The standard to evaluate performance of approximate function of \( l_0 \) norm mainly includes two aspects: noise tolerance and the accuracy of approximation. It is assumed that \( f(x) \) is the approximate function of \( l_0 \) norm, there are two parameters can be used for expressing the property of \( f(x) \). \( \alpha = g(w)_{w=0.5} \) is used for describing the noise tolerance of approximate function and is used for describing the accuracy of function of approximation; in which, \( g(w) = f^{-1}(x) \). When system dimension is higher, \( w_i = (m-1)/m \) (in which, \( m \) is the dimension of signal source) can be selected as the dimension of signal sources. Expectation \( \alpha \) can be adjusted according to noise and the adjustment of \( \beta \) not affected by \( \alpha \) in noise environment.

Up to now, Gaussian function \( f(x) = e^{-x^2/2\sigma^2} \) is used generally as the approximate function [13] of \( l_0 \) norm. For Gaussian function, there is

\[ \alpha = \sqrt{2\sigma^2 \ln 2} \]
\[ \beta = \sigma \sqrt{2 \ln((m-1)/m)} \]

Obviously, both of \( \alpha \) and \( \beta \) depend on \( \sigma \). The adjustment of \( \alpha \) can affect on \( \beta \) inevitably, so that noise tolerance performance of approximate function is reduced.

The disadvantage of one-parameter approximate function can be avoided by selecting approximate function of multi parameters. The paper selects a kind of approximate function of two-parameter that is similar to Sigmoid function:

\[ f_{\sigma,u}(x) = \frac{1}{1 + e^{-u}} \]

For \( f_{\sigma,u}(x) \), there is

\[ \alpha |_{w=0.5} = \sqrt{u}, \quad \beta |_{w=(m-1)/m} = \sigma \sqrt{2 \ln((m-1)/m)} + u \]
Obviously, $\alpha$ only can be determined by parameter $u$, and adjustment of $\beta$ can be achieved through $\sigma$. In other words, the noise tolerance of $f_{\sigma,u}(\alpha)$ and accuracy of approximation can be determined by different parameters.

### 3.2. Blind Source Separation for Measurement Matrix

In practical application, the dimension of measurement matrix is so high that reaches hundreds or thousands. In order to reduce the complexity of calculation and the sensibility to noise, the singular value decomposition for measurement $Y$ is initially needed. The singular value decomposition is designed to decompose the measurement matrix into subspaces of signal and noise, and only retain the subspace of signal, thus transforming the estimation issue of signal DOA into MMV issue which has lower dimension.

By only retaining the subspace of signal, the measured data $\{Y(j)\}_{j=1}^{J}$ shall turn into a $K$ dimension signal, where $K$ is the number of signal source. The singular value decomposition of measurement matrix $Y$ can be expressed as

$$Y = ULV'$$  \hspace{1cm} (7)

Where $Y_{sv}$ is $M \times K$ dimension matrix which including substantially all energy of signal. The $Y_{sv}$ can be expressed as $Y_{sv} = UL_{K} = YVD_{K}$, in which $D_{K} = [I_{K} \ 0]$. Meanwhile, both $S_{sv}$ and $W_{sv}$ can be expressed as $S_{sv} = SVD_{K}$ and $W_{sv} = WVD_{K}$, and then

$$Y_{sv} = \Phi S_{sv} + W_{sv}$$

The formula (8) and formula (2) have the same expression, except the dimension drops from $J$ to $K$, which enables the estimation method of signal DOA which is based on sparse representation to have better instantaneity.

### 3.3. Estimation Method of DOA Constrained by Smooth $l_0$

By using the approximate function of $l_0$ norm defined by formula (5), the sparseness of signal source vector can be expressed as

$$F_{\sigma,u}(S) = N - \sum_{i=1}^{N} f_{\sigma,u}(\|S(i,:)\|_2)$$ \hspace{1cm} (8)

Therefore, if the measured data $Y$ is known, the signal source vector $S$ can be solved by following formula:

$$S(\sigma,u) = \arg \min_{S} \left( L_{\sigma,u}(S) \right)$$, \hspace{1cm} (9)

$$L_{\sigma,u}(S) = -F_{\sigma,u}(S) + \lambda \|Y - \Phi S\|_F^2$$ \hspace{1cm} (10)

Where $u$ is the constant relevant to noise, and $\sigma$ is the number approximate to 0, and $\|A\|_F = \|\text{vec}(A)\|_2$ is the Frobenious norm of matrix $A$. The parameter $\lambda$ controls the sparseness of signal and the compromise of noise level.

As $u$ is in correlation with noise level, it can be considered as a constant here. When $\sigma$ is low, the sparseness of signal source vector $S$ approximate to function $-F_{\sigma,u}(S)$ and has many local minimum points; with the growth of $\sigma$, $f_{\sigma,u}(S)$ shall become smoother and smoother. When $\sigma \to \infty$, there is

$$\lim_{\sigma \to \infty} S_{\sigma,u}(\sigma,u) = \Phi^* \left( \Phi \Phi^* \right)^{-1} Y$$ \hspace{1cm} (3)

Where $\Phi^*$ is the conjugate transposition of matrix $\Phi$. So the formula (10) can be solved by regarding a bigger $\sigma$ value as initial point; then reduce the $\sigma$ value constantly, and then solve
formula (10) until condition of convergence is reached. In order to solve formula (10), the paper adopted an algorithm similar to Gaussian-Newton.

In which, as for fixed \( \sigma, \nu \), the definition for \( G(S) \) is as follows:

\[
G(S) = \begin{bmatrix}
\frac{\chi(1)}{(1 + \chi(1))^2} & 0 & 0 \\
0 & \ddots & 0 \\
0 & 0 & \frac{\chi(N)}{(1 + \chi(N))^2}
\end{bmatrix}
\]

Then there is \( S_{\star}(\sigma, u) = \hat{\xi}(S_{\star}(\sigma, u)) \). Beyond that, for any signal source vector \( S \), there existing actual value scalar \( \gamma \geq 0 \), which shall meet

\[
L_{\sigma, u}(\gamma S) + (1-\gamma)S \leq L_{\sigma, u}(S)
\]

Because of \( S_{\star}(\sigma, u) = \hat{\xi}(S_{\star}(\sigma, u)) \), the answer \( S_{\star}(\sigma, u) \) of signal source vector can be got from formula \( S_{\star}(\sigma, u) = \hat{\xi}(S_{\star}(\sigma, u)) \) by adopting method of iteration for fixed points. To ensure the convergence of algorithm \( L_{\sigma, u}(\xi(S)) \leq L_{\sigma, u}(S) \) must be met. However, not all \( S \) shall meet this, so it can be solved through formula (14). The whole flow of algorithm is as follows:

Initialize:

1. Assume \( S^{(0)} = \Phi^* \left( \Phi \Phi^* \right)^{-1} Y \);
2. Estimate the noise level by method in document, and then determine the parameter \( u \) according to noise level. Assume \( \sigma > \sigma_0 \), repeat:
3. Measure the singular value deposition of matrix \( Y \).
4. Let \( \beta = 1 \);
5. Complete until \( L_{\sigma, u}(\gamma S) + (1-\gamma)S \geq L_{\sigma, u}(S) \quad \beta = \gamma \beta \);
6. \( S^{(i+1)} = \gamma \xi(S^{(i)}) + (1-\gamma)S^{(i)} \)
7. If \( \| S^{(i+1)} - S^{(i)} \| < \eta \sigma \), then \( \sigma = \rho \sigma \);

4. Results and Discussion

This section shall verify the effectiveness of method adopted by this paper by comparing all kinds of different DOA estimation method such as algorithm of Mixed \( l_{2,0} \), \( l_1 - SVD \), MUSIC and CAPON. The estimation of parameter \( \nu \) shall adopt method proposed in document.

The ULA array of array element number \( M=16 \) shall be adopted and the space between array elements is half wavelength of narrow-band signal. Uncorrelated signal is to be considered primarily. The amplitude of narrow-band signal shall meet Gaussian distribution of mean value \( 0 \). Disperse the whole space domain \([0^\circ, 180^\circ]\) into grid point with resolution of \( 1^\circ \), i.e. \( N = 181 \), \( \Phi \in C^{M+N} \). Measure the effect of uncorrelated noise which the signal subjects in time domain and space domain. Firstly select the fixed number of snapshots, like \( J = 100 \). The effect which the number of snapshots made on rebuilding results shall be discussed below.
Figure 2 describes the spatial spectrum got by each of different algorithms when the space between grid points is 1° and 0.5°. The signal to noise ratio $\text{SNR} = 10\,\text{db}$ and the directions of signal source are -10° and 20.7°, and they are not correlated. Figure 2(a) describes the result when the space of grid point is 1°. Although the DOA of the second signal source has not exactly fallen over the grid point, it can be observed that a signal source can be got at 21° from the closest real value of signal source DOA by $JSL_0 - SV$ algorithm. Figure 2(b) describes the result when the space between grid points is 0.5°. It can be observed from the figure that reducing the space between grid points cannot evidently improve rebuilding quality. Figure 2(a) and (b) indicate that all methods can be used to recognize two targets, but sometimes $JSL_0 - SV$ and Mixed $I_{2,0}$ shall receive spurious signal source, especially when SNR is low.

![Figure 2](image)

Figure 2. Estimation of uncorrelated signal DOA: (a) $\text{gird} = 1°$, (b) $\text{gird} = 0.5°$

Figure 3 describes the DOA estimation result when correlation coefficient is 0.95 and arrival angles are 60° and 80° respectively.

From the figure, it can be observed that all methods on the basis of sparseness representation is able to accurately estimate the arrival angle of signal source, but $JSL_0 - SV$ and Mixed $I_{2,0}$ also may get spurious target signal source. If the signal sources number is unknown or SNR is low, $JSL_0 - SV$ shall get spurious signal source; if SNR is low, Mixed $I_{2,0}$ shall get spurious signal source. If the number of snapshots is small, the algorithms of MUSIC and CAPON are unable to effectively estimate the DOA of target signal.

![Figure 3](image)

Figure 3. DOA estimation of correlative signal: (a) $\text{gird} = 1°$, (b) $\text{gird} = 0.5°$
Next we analyze the effect of the number of snapshots on the performance of different DOA estimation algorithms. In the simulation, two strong correlated signals from different directions shall be used with $\text{SNR} = 10\, \text{dB}$, and by changing number of snapshots, the effect of the number of snapshots on algorithm performance can be verified. 500 times of independent simulation shall be done for each snapshot value. If MSE of the simulation result is lower than some fixed threshold, then it shall be deemed as failing to accurately estimate the DOA of signal. Figure 4(a) is the simulation result. Due to the number of snapshots required by estimation method basing on sparse representation is far lower than the number of snapshots required by algorithms of MUSIC and CAPON, the performance curve of algorithms of MUSIC and CAPON has not been drawn in the simulation. The figure shows that the performance of $\text{JS}l_0 - \text{SVD}$ is evidently superior to $\text{Mixed } l_{2,0}$ and $l_1 - \text{SVD}$. Figure 4(b) described the effects of SNR have on estimation performance by each algorithm. In this simulation, the SNR value shall be changed while other values shall retain. Because $l_1 - \text{SVD}$ and $\text{Mixed } l_{2,0}$ shall generate spurious signals when SNR is low, so the performance of is obviously superior to the other two methods. When SNR is high, the performance of and is quite approximate.

![Figure 4](image)

**Figure 4.** Influence of Number of Snapshots and SNR on DOA Estimation Performance:
(a) Number of snapshots on DOA estimation performance
(b) SNR on estimation performance

Finally, we shall compare the computational efficiency by each of different algorithms. For the algorithm of MUSIC is unable to effectively estimate DOA of correlated signal, the simulation shall use uncorrelated signals with fixed SNR and changeable numbers of snapshot, and the result is showed in Figure 1. Although the computational efficiency of algorithm of MUSIC is the highest, it also should be noted that DOA estimation performance by algorithm of MUSIC is for lower than that by method based on sparseness representation. The computational efficiency of $l_1 - \text{SVD}$ and $\text{Mixed } l_{2,0}$ is lower than $\text{JS}l_0 - \text{SVD}$. This is because the efficiency of two-level cone planned algorithm of sparseness representation issue constrained by resolution $l_1$ is low, and also because is seeking for sparseness solution in the whole space and the computational efficiency is low when the dimension of signal is high.

<table>
<thead>
<tr>
<th>Number of Snapshots</th>
<th>$\text{JS}l_0 - \text{SVD}$</th>
<th>$\text{Mixed } l_{2,0}$</th>
<th>$l_1 - \text{SVD}$</th>
<th>MUSIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>0.047</td>
<td>0.172</td>
<td>3.484</td>
<td>~</td>
</tr>
<tr>
<td>100</td>
<td>0.062</td>
<td>0.218</td>
<td>4.058</td>
<td>~</td>
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<td>150</td>
<td>0.078</td>
<td>0.406</td>
<td>4.453</td>
<td>0.028</td>
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<tr>
<td>200</td>
<td>0.094</td>
<td>0.703</td>
<td>6.086</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Table 1. Comparison of computational efficiency
5. Conclusion
This paper transforms the estimation issue of signal DOA into a solution issue combined with sparseness representation. By the singular value decomposition of received data matrix, the combination of numbers of snapshots of each time and frequency has been achieved; then the estimation of signal source DOA has been realized by solving a combined optimization issue constrained by smooth $l_0$ norm sparseness. The signal DOA estimation method based on sparseness representation not only can reduce data size effectively, but also has following merits: better anti-noise performance; higher computational efficiency; applicable to correlated and uncorrelated signals. By comparing with other DOA estimation methods, the effectiveness and superiority of methods in this paper has been proved. In addition to this, $JSL_0 - SVD$ also has some merits as follows: better anti-noise performance; higher computational efficiency; applicable to correlated and uncorrelated signals.

References