Study on outage performance gap of two destinations on CR-NOMA network

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ABSTRACT

Non-orthogonal multiple access (NOMA) and cognitive radio (CR) are promising to overcome spectral scarcity problem encountered in applications implementations in wireless communication. Especially, massive connectivity in such network is strict requirement in network deployment. This study aims to improve spectral efficiency at two secondary destinations by investigating a CR-NOMA network under situation of the perfect successive interference cancellation (SIC). We also derive the exact outage probability for secondary users. Furthermore, an approximate computation method is applied to indicate more insights. It is confirmed that the performance achieved together with performance gap among two users can be obtained due to different power allocation factors assigned to users.

Keywords:
Cognitive radio
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1. INTRODUCTION

The spectral efficient and energy-efficient requirements are necessary to satisfy the explosive increase of mobile user in wireless system with high-rate services. However, high spectral efficiency (SE) cannot be achieved since the fixed spectrum allocation strategy is adopted. Unfortunately, 30 percentages of the licensed spectrum in the United States is fully occupied as the report from the Federal Communications Commission [1]. By allowing the primary network to share its frequency band with the secondary network, cognitive radio (CR) has been studied and hence SE improvement achieved [2]. In principle of CR, spectrum sharing paradigm permits the secondary users (SUs) to operate together with the primary users (PUs) at the same band and power constraint must be obeyed to limit interference impact caused by the PUs [3, 4]. Several techniques such as cellular networks, relay networks, and wireless sensor networks, benefit from implementation of CR to provide the potential SE improvement.

To further provide massive connectivity, more advantages can be achieved by employing multiple access for mobile users. In particular, the network allocates resource to users by dividing the total radio resources with two underlying techniques, i.e. orthogonal multiple access (OMA) and non-orthogonal multiple access (NOMA). The interference can be eliminated in OMA scheme while NOMA employs successive interference cancellation (SIC) technique to alleviate interference from other users’ signal [5]. By exploiting the users’ channel asymmetry, NOMA can remarkably enhance the SE and then the transmission latency can be reduced [5-8]. The authors in [9] showed that the achievable rate region in the uplink NOMA is improved in comparison with OMA and such analysis is adopted in wireless powered communication...
(WPC) networks. In [10], main results reported that NOMA with advantage of improved user fairness and it provide more benefits compared to OMA. It is further proved that NOMA performs better than OMA in both downlink and uplink by achieving the problem of joint maximization of the downlink/uplink rates while taking fairness between users is satisfied [11]. In [12], the authors presented energy efficiency in wireless powered NOMA networks and system performance is evaluated. In addition, recent works [13-22] considered advantage of NOMA to implement in emerging networks. In particular, this paper develops system based on results in [23-25]. More specifically, in this paper, we formulate the received signal at the secondary user (SU) which can extract the data signal by using SINR or SNR.}

2. SYSTEM MODEL

We assume that the system model with a downlink dual-hop underlay cognitive radio–non-orthogonal multiple access (CR-NOMA) network shown in Figure 1, in which there are a primary destination ($P_D$) who is located in primary network (PN), a secondary source ($BS$), a relay ($R$) operating in half-duplex mode and two destination users ($U_1$; $U_2$). The wireless channels follow Rayleigh fading-channel $u$ with channel gain $\Omega_u$. These channels assigned as in Figure 1 are $h_0$, $h_1$, $g_1$ and $g_2$, are independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables (RVs). Single antenna is assumed at each node. In this scenario, a perfect channel state information (CSI) is adopted. As Figure 1, the distances between nodes are denoted by $h_0$, $h_1$, $g_1$ and $g_2$. In CR-NOMA, the BS make interference to $P_D$. It is noted that $R$ requires decode-and-forward (DF) mode to forward signal to far users. It is assumed that $R$ is placed very far from the transmit primary source $P_D$ and hence it cannot interfere with the primary network as shown in Figure 1. The power constraint for operations of both primary network and secondary network is considered in this context.

![Figure 1. NOMA in cognitive radio network](image)

The transmit power at secondary source is set based on constraint as above consideration

$$P_{BS} \leq \min \left( I \frac{1}{|h_0|^2}, \bar{P}_{BS} \right)$$  \hspace{1cm} (1)

where $\bar{P}_{BS}$ and $I$ is denoted as the maximum average transmit power available at $BS$ and interference temperature constraint (ITC) at $P_D$, respectively. We call $a_1, a_2$ as power allocation factors. In the first time slot, $R$ received the following signal

$$y_R(k) = h_1 [\sqrt{P_{BS}a_1}s_1(k) + \sqrt{P_{BS}a_2}s_2(k)] + n_R(k)$$  \hspace{1cm} (2)

where $h_0 \sim CN(0, \Omega_{h0}), h_1 \sim CN(0, \Omega_{h1}), n_R \sim CN(0, \sigma_R^2)$, it is assumed that $a_1 > a_2$ and $a_1 + a_2 = 1$.

By using NOMA, to detect signal $s_1$ $R$ decodes and removes $s_2$ from the received signal. Therefore, it need be determined the signal-to-interference-plus noise ratio (SINR) and signal-to-noise ratio (SNR) to detect $s_1$ and $s_2$ at $R$ as follows

$$\gamma_{R,s_1} = \frac{\rho_{BS}a_1|h_1|^2}{\rho_{BS}a_2|h_1|^2+1}$$  \hspace{1cm} (3)
where $\rho_{BS} = \frac{P_{BS}}{\sigma^2_R}$

$$y_{R,2} = \rho_{BS} a_2 |h_1|^2$$

(4)

Then, within the second slot, $R$ forwards the detected superimposed signal $\sqrt{P_R} a_2 \tilde{s}_1(k) + \sqrt{P_R} a_2 \tilde{s}_2(k)$, where $P_R$ is the transmitted power at $R$, $\tilde{s}_1(k)$ and $\tilde{s}_2(k)$ are the detected and forwarded data to the respective receivers. Therefore, $U_i$ receives the following signal:

$$y_{RU_i}(k) = g_i \left[ \sqrt{P_R} a_1 \tilde{s}_1(k) + \sqrt{P_R} a_2 \tilde{s}_2(k) \right] + n_{RU_i}(k)$$

(5)

where $i \in \{1, 2\}$, $g_i \sim \mathcal{CN}(0, \Omega_{g_i})$ and $n_{R} \sim \mathcal{CN}(0, \sigma^2_R)$. Furthermore, $U_2$ implements SIC by detecting $\tilde{s}_1(k)$ while considering its own data $\tilde{s}_2(k)$ as a noise. The SINR of which can be written as:

$$y_{RU_2,s_1} = \frac{\rho_R a_1 |g_1|^2}{\rho_R a_2 |g_2|^2 + 1}$$

(6)

where $\rho_R = \frac{P_R}{\sigma^2_{RU_1}}$. Then, by alleviate interference existing in (6) it can be detected the remaining signal. Therefore, to detects its own signal at $U_2$, SNR is given by

$$y_{RU_2,s_2} = \rho_R a_2 |g_2|^2$$

(7)

It is worth noting that $U_1$ is allocated with higher power factor, $s_1$ has higher priority to detect compared with remaining signal, then SINR is expressed by

$$y_{RU_1,s_1} = \frac{\rho_R a_1 |g_1|^2}{\rho_R a_2 |g_2|^2 + 1}$$

(8)

3. PERFORMANCE ANALYSIS AND NUMERICAL RESULTS

3.1. Outage probability analysis at user 1

In this section, we examine the outage probability (OP) for $s_1$ and $s_2$. In [10-13], the OP of a signal is defined as the probability that the achievable rate is below than a predefined rate threshold $R_{th}$, i.e., $P_{U_1} = P[R < R_{th}]$. Therefore, the OP of $s_1$ can be derived as:

$$P_{U_1} = \Pr \left( \min \{ y_{R,s_1}, y_{RU_1,s_1} \} < y_1 \right) = 1 - \Pr \left( y_{R,s_1} > y_1, y_{RU_1,s_1} > y_1 \right)$$

$$= 1 - \Pr \left( \frac{\rho_{BS} a_1 |h_1|^2}{\rho_{BS} a_2 |h_1|^2 + 1} > y_1, \frac{\rho_{BS} a_2 |g_1|^2}{\rho_{BS} a_2 |g_1|^2 + 1} > y_1, \rho_{BS} < \frac{\rho_y}{|h_1|^2} \right)$$

$$+ \Pr \left( \frac{\rho_{BS} a_1 |h_1|^2}{\rho_{BS} a_2 |h_1|^2 + |h_0|^2} > y_1, \frac{\rho_{BS} a_2 |g_1|^2}{\rho_{BS} a_2 |g_1|^2 + |h_0|^2} > y_1, \rho_{BS} > \frac{\rho_y}{|h_0|^2} \right)$$

(9)

where $\rho_y = \frac{1}{\sigma^2_D}$ and $y_1 = 2^{2R_1} - 1$ is SNR related to interference and SNR related to target rate $R_1$ of user $U_1$ respectively. Based on distribution functions of wireless channels, it can be expressed as:

$$A_1 = \Pr \left( |h_1|^2 > \frac{y}{\rho_{BS}^2}, |g_1|^2 > \frac{y}{\rho_R}, |h_0|^2 < \frac{\rho_y}{\rho_{BS}} \right) = \frac{\gamma}{\rho_{BS}} \int_{0}^{\gamma} f_{h_1}(x) dx \int_{0}^{\gamma} f_{h_2}(y) dy \int_{0}^{\gamma} f_{g_1}(z) dz$$

$$= e^{-\frac{\rho_{BS}}{\rho_y |h_0|^2}} \left( 1 - e^{-\frac{\rho_y}{\rho_{BS} |h_0|^2}} \right)$$

$$= e^{-\frac{\rho_{BS}}{\rho_y |h_0|^2}} \left( 1 - e^{-\frac{\rho_y}{\rho_{BS} |h_0|^2}} \right)$$

$$= e^{-\frac{\rho_{BS}}{\rho_y |h_0|^2}} \left( 1 - e^{-\frac{\rho_y}{\rho_{BS} |h_0|^2}} \right)$$

(10)

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where $\psi = \gamma_2$. In similar way, it can be computed the second part of (9) as:

$$A_1 = \Pr \left( |h_1|^2 > \psi |h_1|^2 |g_1|^2 \right) = \int f_{|h_1|^2} (x) \int f_{|g_1|^2} (y) f_{|h_1|^2} (z) dy dz$$

$$= \int \frac{1}{\Omega y} e^{-\frac{x^2}{\Omega y}} dx \int \frac{1}{\Omega x} e^{-\frac{y^2}{2 \Omega x}} dy = \frac{\rho_f \Omega_{\psi}}{\rho_f \Omega_{\psi} + \psi} e^{-\frac{\rho_f}{\rho_f \Omega_{\psi} + \psi} \frac{1}{2 \Omega x}}$$

by replacing (9) by (10) and (11), (9) can be re-expressed as:

$$\mathcal{P}_{U_1} = 1 - \left[ e^{-\frac{\psi}{\rho_f \Omega_{\psi} \Omega_{\psi} \Omega_{\psi}}} \left( 1 - e^{-\frac{\rho_f}{\rho_f \Omega_{\psi} \Omega_{\psi} \Omega_{\psi}}} + \frac{\rho_f \Omega_{\psi}}{\rho_f \Omega_{\psi} + \psi} e^{-\frac{\rho_f}{\rho_f \Omega_{\psi} + \psi} \frac{1}{2 \Omega x}} \right) \right]$$

it is noted that the above formula is correct when $a_1 > a_2$.

3.2. Outage probability analysis if perfect SIC at user 2

Similar to the signal $s_1$, at user $U_2$, the OP of the signal $s_2$ can be expressed as:

$$\mathcal{P}_{U_2} = \Pr \left( \min \left( \gamma_{R_2}, \gamma_{R_2 I} \right) < \gamma_2 \right) = 1 - \Pr \left( \gamma_{R_2 I} > \gamma_2 \right)$$

$$= 1 - \left[ \Pr \left( \frac{\rho_f a_2}{|h_2|^2} > \gamma_2, \frac{\rho_f a_2}{|h_2|^2} > \gamma_2, \frac{\rho_f}{\rho_f \Omega_{\psi} + \psi} \right) \left( \frac{\rho_f}{|h_2|^2} < \frac{\rho_f}{\rho_f \Omega_{\psi} + \psi} \right) \right]$$

where $\gamma_2 = 2^R_2 - 1$ with $R_2$ corresponding target rate of $U_2$. The first part and the second part of (13) can be further computed by:

$$B_2 = \Pr \left( |h_2|^2 > \frac{\gamma_2}{\rho_f a_2}, |g_2|^2 > \frac{\gamma_2}{\rho_f a_2}, |h_2|^2 < \frac{\rho_f}{\rho_f \Omega_{\psi} + \psi} \right)$$

$$= \int \frac{f_{|h_2|^2} (x)}{\rho_f} \int \frac{f_{|g_2|^2} (y)}{\rho_f} dy \int \frac{f_{|h_2|^2} (z)}{\rho_f} dz = e^{-\frac{\gamma_2}{\rho_f a_2}} e^{-\frac{\gamma_2}{\rho_f a_2}} \left( 1 - e^{-\frac{\rho_f}{\rho_f \Omega_{\psi} \Omega_{\psi} \Omega_{\psi}}} \right)$$

then, other term can be given as:

$$B_2 = \Pr \left( |h_2|^2 > \frac{\gamma_2}{\rho_f a_2}, |g_2|^2 > \frac{\gamma_2}{\rho_f a_2}, |h_2|^2 > \frac{\rho_f}{\rho_f \Omega_{\psi} + \psi} \right)$$

$$= \int \frac{f_{|h_2|^2} (x)}{\rho_f} \int \frac{f_{|g_2|^2} (y)}{\rho_f} dy \int \frac{f_{|h_2|^2} (z)}{\rho_f} dz = e^{-\frac{\gamma_2}{\rho_f a_2}} e^{-\frac{\gamma_2}{\rho_f a_2}} \left( 1 - e^{-\frac{\rho_f}{\rho_f \Omega_{\psi} \Omega_{\psi} \Omega_{\psi}}} \right)$$

by substituting (15) and (14) into (13), (13) can be rewritten as:
\[
\mathcal{P}^{\text{P-SIC}}_{U_2} = 1 - \left[ e^{-\frac{\gamma_1}{\rho_{BS}^2 h_1^2 \rho_{2}^2}} \left( 1 - e^{-\frac{1}{\rho_{BS}^2 h_1^2 \rho_{2}^2}} \right) + \frac{\rho_{2} \alpha_1 \alpha_2}{\rho_{2} \alpha_1 \alpha_2 + \rho_{2} \alpha_1 \alpha_2} e^{-\frac{\gamma_1}{\rho_{BS}^2 h_1^2 \rho_{2}^2}} \right]
\]

### 3.3. Outage analysis if imperfect SIC at user 2

The SINR and signal-to-noise ratio (SNR) of decoding \( s_2 \) at \( R \) and at destination \( U_2 \) can be respectively written as:

\[
\gamma_{R,s_2} = \frac{\rho_{BS} a_2 |h_1|^2}{\rho_{BS} |f_1|^2 + 1}
\]

\[
\gamma_{RU_2,s_2} = \frac{\rho_{BS} a_2 |g_2|^2}{\rho_{BS} |f_2|^2 + 1}
\]

then, the OP in case of imperfect SIC at \( U_2 \) can be calculated by:

\[
\mathcal{P}^{\text{P-SIC}}_{U_2} = \mathbb{P} \left( \min \left( \gamma_{R,s_2}, \gamma_{RU_2,s_2} \right) < \gamma_2 \right)
= 1 - \mathbb{P} \left( \gamma_{R,s_2} > \gamma_2, \gamma_{RU_2,s_2} > \gamma_2 \right)
= 1 - \left[ \mathbb{P} \left( \frac{\rho_{BS} a_2 |h_1|^2}{\rho_{BS} |f_1|^2 + 1} > \gamma_2, \frac{\rho_{BS} a_2 |g_2|^2}{\rho_{BS} |f_2|^2 + 1} > \gamma_2, \tilde{\rho}_{BS} < \frac{\rho_{1}}{\rho_2} \right) \right]
+ \mathbb{P} \left( \frac{\rho_{BS} a_2 |h_1|^2}{\rho_{BS} |f_1|^2 + 1} > \gamma_2, \frac{\rho_{BS} a_2 |g_2|^2}{\rho_{BS} |f_2|^2 + 1} > \gamma_2, \tilde{\rho}_{BS} > \frac{\rho_{1}}{\rho_2} \right)
\]

(19)

similarly, (19) can be rewritten as:

\[
\mathcal{P}^{\text{P-SIC}}_{U_2} = 1 - \left[ \left( 1 - e^{-\frac{\rho_1}{\rho_{BS}^2 a_1^2}} \right) + \left( \frac{\rho_2}{\rho_{BS}^2 a_1^2} e^{\frac{\gamma_1}{\rho_{BS}^2 a_1^2}} + \frac{\rho_2}{\rho_{BS}^2 a_1^2} e^{\frac{\gamma_2}{\rho_{BS}^2 a_1^2}} \right) \right]
\]

(20)

### 3.4. Asymptotic analysis

This part provides approximate performance as extra insights in our considered system. When \( \rho \to \infty \), it can be applied \( e^{-x} \approx 1 - x \), then approximate performance can be archived as below. The approximate OP of user \( U_1 \) can be given by:

\[
\mathcal{P}^{\text{asym,} U_1} = 1 - \left[ \left( 1 - \frac{1}{\rho_{BS}^2 a_1^2} \right) + \left( \frac{\rho_1}{\rho_{BS}^2 a_1^2} e^{\frac{\gamma_1}{\rho_{BS}^2 a_1^2}} + \frac{\rho_2}{\rho_{BS}^2 a_1^2} e^{\frac{\gamma_2}{\rho_{BS}^2 a_1^2}} \right) \right]
\]

(21)

the approximate OP of user \( U_2 \) in case of perfect SIC can be given by:

\[
\mathcal{P}^{\text{asym,} U_2} = 1 - \left[ \left( 1 - \frac{\rho_1}{\rho_{BS}^2 a_1^2} e^{\frac{\gamma_1}{\rho_{BS}^2 a_1^2}} + \frac{\rho_2}{\rho_{BS}^2 a_1^2} e^{\frac{\gamma_2}{\rho_{BS}^2 a_1^2}} \right) \right]
\]

(22)

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the approximate OP of user $U_2$ in case of imperfect SIC can be formulated by:

$$P_{\text{asymp},U_2}^{\text{SOC}} = 1 - \frac{\rho_2}{\rho_2 a_2} \left( 1 - \frac{\gamma_2}{\Omega_{h_2} a_2} - \frac{\gamma_2}{\rho_2 a_2} \right) \left( \frac{\gamma_2 \Omega_{h_2}}{\Omega_{h_2} a_2} + 1 \right) \left( \frac{\gamma_2 \Omega_{f_2}}{1 + \rho_2 a_2} + 1 \right)^{-1}$$

$$+ \frac{\Omega_{g_2} \gamma_2}{\rho_2 a_2} + 1 \right)^{-1} \left( \frac{\gamma_2 \Omega_{f_2}}{a_2} + 1 \right)^{-1} \left( \frac{\gamma_2 \Omega_{h_2} \rho_2 + \rho_2 a_2} {\gamma_2 \Omega_{h_2} a_2} \right)^{-1}$$

$$\times \rho_2 a_2 \Omega_{h_2} \left( 1 - \frac{\rho_2}{\rho_2 a_2} - \frac{\gamma_2}{\rho_2 a_2} - \frac{\gamma_2}{\rho_2 a_2} \right)$$

(23)

3.5. Throughput

In term of throughput, each user can be shown throughput performance as:

$$\tau_{U*} = (1 - P_{U*}) R_*$$

(24)

where $* \in \{1, 2\}$.

4. NUMERICAL RESULTS

In this section, we evaluate the performance of CR-NOMA, we set power allocation factors $a_1 = 0.8$ and $a_2 = 0.2$, the target rate is set to be $R_1 = 1$ and $R_2 = 1.5$, the channel gains $\Omega_{h_0} = 1, \Omega_{h_1} = 1, \Omega_{g_1} = 1, \Omega_{g_2} = 0.4, \Omega_{f_1} = \Omega_{f_2} = 0.001$. Interference between PN and SNR is $\rho_1 = 40$ dB. Figure 2 and Figure 3 plot the OP of two secondary destinations, as varying interference level $\rho_1$ and power allocation factor, transmit SNR. Outage performance of $U_1$ is better than that of $U_2$. It can be seen that when higher transmit SNR is required, outage performance will be improved significantly at considered range of SNR and OP meets saturation trend as SNR is from $50$ (dB) to $60$ (dB).

The asymptotic curves match with the analytical curves very well at high SNR. This output confirms exact approximate expressions of outage probability archived for two users. It is intuitively seen that no ITC case exhibits lowest performance since no harmful interference from the PN exists. It can be seen performance gap of these cases with different data rate is small, it exhibit acceptance performance for such NOMA with acceptable small value of target rate. In addition, Monte-Carlo simulation results match with analytical results very well in whole range of SNR. Figure 4 proved that higher rate result in worst case of outage performance. In addition, as observation from Figure 5, throughput is high at high SNR and high $\rho_1$. 

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![Figure 2. Outage performance versus SNR at secondary source](image2.png)

![Figure 3. Impact of ITC on outage performance versus SNR at secondary source](image3.png)
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5. CONCLUSION

In this paper, CR-NOMA networks over Rayleigh fading channels is studied by exploring the end-to-end closed-form expressions to indicate outage performance. To compare the outage performance of two secondary destinations, we derived expressions of outage probability and then numerical results are provided performance comparisons of two users in CR-NOMA network. As main result, the fairness of two users is satisfied as in numerical results by the proper selection of power allocation factors. Other condition is that interference to primary network can be constrained. Moreover, comparison results of the outage behavior showed that $U_1$ performs better than $U_2$ in considered scenarios. Finally, in the future work, we will consider multiple users who operate in manner of CR-NOMA network.

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