Tractable computation in outage performance analysis of relay selection NOMA

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ABSTRACT
In recent years, using full-duplex (FD) transmission model provides enhanced bandwidth efficiency and improved performance for non-orthogonal multiple access (NOMA) system. However, lack of papers have investigated FD relay together with relay selection issue to improve performance of NOMA system. The problems in power allocation for two NOMA users satisfying fairness as well as relay selection strategy are studied in this paper. By considering the outage performance of proposed scheme with its vital result, general NOMA wireless networks can be developed for future networks due to its improved performance. Simulation results show that the relaying selection scheme can achieve a significant performance improvement by increasing required quantity of relay.

1. INTRODUCTION
In recent years, non-orthogonal multiple access (NOMA) has attracted a lot of research works due to increasing number of connections. Furthermore, it is considered as one of the modern technologies applied in 5th generation (5G) mobile networks in order to considerably enhance the system spectral efficiency for 5G communication networks [1-4]. In NOMA, multiple users have their signals which are incorporated into mixed signal in power domain. As a result, these users in NOMA are served in the same domain such as time, frequency or code domain. To overcome bad channel condition, NOMA procedure mainly allocates greater transmit power dedicated for users suffering from weak channel conditions. In this scenario, by treating others’ signal as noise, these weak users can detect and decode its symbols precisely due to greater power level distributed for far user. On the other hand, based on the successive interference cancellation (SIC) technique, users which have strong channel conditions are capable to detect its own signal. In addition, as compared to the OMA system, there is a significant improvement in NOMA throughput performance [2, 5]. Recently, cooperative networks are introduced as in [6, 7], the author focused on system evaluation related to processing signal at relay and outage probability is guaranteed. Fortunately, such cooperative schemes can be combined with NOMA to reform cooperative NOMA. In particular, it need be investigated performance and it can be enhanced by employing NOMA scheme into system model. In principle, by utilizing SIC, the received message is decoded at the receiver while the transmitter treating interference signal and such NOMA is proposed in NOMA technique as in [8]. Outage probability is main metric to consider in cooperative NOMA network where the message transmission is assisted over an amplify-and-forward (AF) relay from the source node [9]. Moreover, in [10], to increase the system
performance NOMA scenario is applied and the mathematical formula in closed form of probability has also been found. In [11], performance of randomly-deployed subscribers of a downlink network are performed in terms of system outage probability and the achievable rate.

To improve the outage performance of all devices in the relay selection (RS) system, the authors in [12] have proposed a two stage RS in cooperative NOMA model. Furthermore, it is similar to [12] that in order to improve system performance, a full-duplex scheme is required to employ in NOMA as considered model by the authors in [13]. Motivated by results in [14-25], this paper targets the concentration on cooperative NOMA integrated with relay selection model and full-duplex scheme employed at relay nodes by inspired by these techniques. In particular, outage and throughput performance are investigated. As main contribution of this paper, we derive the closed-form expression in term of outage event to evaluate system performance.

2. SYSTEM MODEL

We consider a downlink system model is shown as Figure 1, where a base station (BS) would like to send a message to the \( N \) near user (NU) so-called as relay (R) to broadcast BS’s data signal to a pair of far users (FU) that set as device 1 (D1) and device 2 (D2). More specifically, Figure 1 illustrates the full-duplex relay in two hop NOMA, which contains one base station (BS), two near NOMA user (strong user) that is capable of the full-duplex transmission, and far NOMA users (weak user). In this model, we denote \( D_{n} \), \( n=1,2 \) which exhibit performance gap. The BS is a single antenna transmission source while the NUs are designed with two separated antennas for full-duplex transmission. As a result, self-interference (SI) due to two antennas scheme still exists. We assume that there does not exist a direct link from the BS to FU, \( h_{n} \) and \( g_{n,i} (i=1,2) \) are denoted the Rayleigh fading channel coefficients of the link BS-NU and NU-FU, respectively. The random variables \( |h_{n}|^2 \) and \( |g_{n,i}|^2 \) are follow the exponential distribution with parameters \( \lambda_{hn} \) and \( \lambda_{gi} (i=1,2) \), respectively. Following principle of normal NOMA, users are distributed in order based on the channel conditions. As show in Figure 1, we assumed that D1 and D2 are used for different data transmission, in which D1 is used for low speed applications, and vice versa for D2 which served high speed data rate.

![Figure 1. System model of relay selection NOMA with full-duplex transmission](image)

The transmission between the BS-NU and NU-FU are divided into two phases. In the early phase, the BS sends the superposition signal, \( a_{x_{1}}x_{1} + a_{x_{2}}x_{2} \) with power allocation factors satisfying \( a_{x_{1}}^2 + a_{x_{2}}^2 = 1 \), where \( x_{1} \) and \( x_{2} \) denotes the message for BS sent to intermediate devices and then these signal intend to transmit to D1 and D2, \( a_{x_{i}}^2 \) denotes the power allocation factor for message \( x_{i} \). In this FD scheme, \( x_{SI} \) denotes self-interference signal. In such case, the residual loop self-interference (SI) is set as a Rayleigh fading feedback channel with the coefficient \( h_{SI} \), and random variables \( |h_{SI}|^2 \) is also follows the exponential
distribution with mean value $\lambda_{\text{in}}$. Because of it must be satisfied quality-of-service (QoS) requirements, it is assumed that $a_1 > a_2$.

Therefore, the received signal at the NU user, i.e., $n$-th relay, $1 \leq n \leq N$ can be given as:

$$y_n' = \sqrt{P_5} h_n (a_1 x_1 + a_2 x_2) + \sqrt{P_5} h_{3n} x_{3n} + n_r,$$

(1)

where $P_5$ denote the transmission power at the source, $n_r$ stands for the additive Gaussian noise (AWGN) at $n$-th NU user with zero mean and variance $N_0$.

It need be computed SNR at the relay to detect signal of $x_1$:

$$\gamma_{SR1,n} = \frac{P_5 a_1^2 |h_n|^2}{P_5 a_1^2 |h_n|^2 + \alpha P_5 |h_{3n}|^2 + N_0}.$$  

(2)

Then, SNR is computed to decode signal $x_2$ after requiring SIC procedure:

$$\gamma_{SR2,n} = \frac{P_5 a_2^2 |h_n|^2}{\alpha P_5 |h_{3n}|^2 + N_0}.$$  

(3)

We call $P_5$ as transmit power at the selected relay, $n_{D_i}$ denotes the AWGN at $D_i$, i.e., $n_{D_i} \sim CN(0, N_0)$. In this case, the received signal at far user $D_1$, $D_2$ are given as following equation:

$$y_{D_i} = \sqrt{P_6} g_{n,i} (a_1 x_1 + a_2 x_2) + n_{D_i}, i \in \{1, 2\}.$$  

(4)

We then compute SNR at user $D_1$ to attain signal $x_1$:

$$\gamma_{RU1,n} = \frac{P_6 a_1^2 |g_{n,1}|^2}{P_6 a_1^2 |g_{n,1}|^2 + N_0}.$$  

(5)

We then consider SNR at user $D_2$ to eliminate interference signal $x_1$ and it can be computed by:

$$\gamma_{RU2,n} = \frac{P_6 a_2^2 |g_{n,2}|^2}{P_6 a_2^2 |g_{n,2}|^2 + N_0}.$$  

(6)

Similarly, at relay, SIC is performed at user $D_2$ to decode its own signal $x_2$:

$$\gamma_{RU2,n} = \frac{P_6 a_2^2 |g_{n,2}|^2}{N_0}.$$  

(7)

Then, we consider on criteria to relay selected to signal forwarding to far users. In particular, the index of best relay $n^1*$ is selected to provide a maximum end-to-end SNR among the BS-NU-D1 links as:

$$n^1* = \arg\max_{n=1,2,\ldots} \min (\gamma_{SR1,n}, \gamma_{RU1,n}).$$  

(8)

Similarly, the index of best relay $n^2*$ is selected to provide a a maximum end-to-end SNR among the S-NU-D2 links as:
3. SYSTEM PERFORMANCE ANALYSIS

3.1. Performance of user D1

In the scenario of FD mode, we provide following results:

Proposition 1: The closed-form expression of outage probability at user D1

\[
OP_{D1}^{FD} = \min_{n=1,...} \left\{ \Pr \left( \gamma_{SR_{1,n}}^{FD} < \lambda_{D1}^{FD} \cap \gamma_{RU_{1,n}}^{FD} < \lambda_{D1}^{FD} \right) \right\}
\]

\[
= \prod_{n=1}^{N} \left( 1 - \frac{1}{\gamma_{n,D1}^{FD} \cdots \gamma_{n,D1}^{FD}} \right)
\]

(10)

Proof:

In which, it can be computed components as:

\[
A = \Pr \left( \gamma_{SR_{1,n}}^{FD} \geq \lambda_{D1}^{FD}, \gamma_{RU_{1,n}}^{FD} \geq \lambda_{D1}^{FD} \right)
\]

\[
= \Pr \left( P_R a_1^2 | h_n |^2 + P_R a_2^2 | h_n |^2 + \sigma P_R | h_n |^2 + N_0, P_U a_1^2 | g_{n,1} |^2 \geq \lambda_{D1}^{FD} \left( P_R a_1^2 | g_{n,1} |^2 + N_0 \right) \right)
\]

(11)

Then it is rewritten as:

\[
A = \Pr \left( | h_n |^2 \geq \frac{\lambda_{D1}^{FD} \sigma P_R | h_n |^2 + \lambda_{D1}^{FD} N_0}{P_R (a_1^2 - \lambda_{D1}^{FD} a_2^2)} \right) \times \Pr \left( | g_{n,1} |^2 \geq \frac{\lambda_{D1}^{FD} N_0}{P_U (a_1^2 - \lambda_{D1}^{FD} a_2^2)} \right)
\]

(12)

In this case, we have two calculations as below:

\[
A_1 = \Pr \left( | h_n |^2 \geq \frac{\lambda_{D1}^{FD} \sigma P_R | h_n |^2 + \lambda_{D1}^{FD} N_0}{P_R (a_1^2 - \lambda_{D1}^{FD} a_2^2)} \right) \text{ if } a_1^2 > \lambda_{D1}^{FD} a_2^2
\]

\[
= \exp \left( - \frac{\lambda_{D1}^{FD} N_0}{P_R (a_1^2 - \lambda_{D1}^{FD} a_2^2) \lambda_{mn}} \right) \frac{P_R (a_1^2 - \lambda_{D1}^{FD} a_2^2) \lambda_{mn}}{P_R (a_1^2 - \lambda_{D1}^{FD} a_2^2) \lambda_{mn}}
\]

(13)

and

\[
A_2 = \Pr \left( | g_{n,1} |^2 \geq \frac{\lambda_{D1}^{FD} N_0}{P_U (a_1^2 - \lambda_{D1}^{FD} a_2^2) \lambda_{pm,1}} \right) = \exp \left( - \frac{\lambda_{D1}^{FD} N_0}{P_U (a_1^2 - \lambda_{D1}^{FD} a_2^2) \lambda_{pm,1}} \right)
\]

(14)

Then \( A \) is rewritten as:

\[
A = \exp \left( - \frac{\lambda_{D1}^{FD} N_0}{P_R (a_1^2 - \lambda_{D1}^{FD} a_2^2) \lambda_{mn}} \right) \frac{P_R (a_1^2 - \lambda_{D1}^{FD} a_2^2) \lambda_{mn}}{P_R (a_1^2 - \lambda_{D1}^{FD} a_2^2) \lambda_{mn}} \exp \left( - \frac{\lambda_{D1}^{FD} N_0}{P_U (a_1^2 - \lambda_{D1}^{FD} a_2^2) \lambda_{pm,1}} \right)
\]

(15)

Therefore, such outage probability for user D1 is given by:
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\[
OP_{1}^{FD} = \prod_{i=1}^{\infty} \left(1 - \frac{P_{c}(a_{i}^{2} - \lambda_{i}^{FD} a_{i}^{2}) \lambda_{n}^{FD}}{\lambda_{i}^{FD} \alpha P_{c} \lambda_{m} + P_{c}(a_{i}^{2} - \lambda_{i}^{FD} a_{i}^{2}) \lambda_{m}^{FD}} \right) \times \exp \left( - \frac{\lambda_{i}^{FD} N_{0}}{P_{c}(a_{i}^{2} - \lambda_{i}^{FD} a_{i}^{2}) \lambda_{m}^{FD}} - \frac{\lambda_{i}^{FD} N_{0}}{P_{c}(a_{i}^{2} - \lambda_{i}^{FD} a_{i}^{2}) \lambda_{m}^{FD}} \right)
\]  

(16)

where \( \lambda_{i}^{FD} = 2^{R} - 1 \).

Similarly, in HD mode, such outage performance can be shown as:

\[
OP_{1}^{HD} = \prod_{i=1}^{\infty} \left(1 - \frac{\lambda_{i}^{HD} N_{0}}{P_{c}(a_{i}^{2} - \lambda_{i}^{HD} a_{i}^{2}) \lambda_{m}^{HD} - P_{c}(a_{i}^{2} - \lambda_{i}^{HD} a_{i}^{2}) \lambda_{m}^{HD} \lambda_{m,2}} \right) \). 

(17)

where \( \lambda_{i}^{HD} = 2^{R} - 1 \).

3.2. Performance of user D2

To consider performance of user of D2, the related outage probability can be given by:

Proposition 2: The closed-form expression of outage probability at user D2 can be given by:

\[
OP_{2}^{FD} = \min_{n_{1},n_{2}} \left\{ \Pr \left( \gamma_{SR2,n_{1}} \geq \lambda_{2}^{FD}, \gamma_{BU12,n_{2}} \geq \lambda_{2}^{FD}, \gamma_{BU2,n_{2}} \geq \lambda_{2}^{FD} \right) \right\} \]

\[
= \prod_{i=1}^{\infty} \left(1 - \prod_{j=1}^{\infty} \frac{\lambda_{j}^{FD} N_{0}}{P_{c}(a_{j}^{2} - \lambda_{j}^{FD} a_{j}^{2}) \lambda_{m}^{FD} - P_{c}(a_{j}^{2} - \lambda_{j}^{FD} a_{j}^{2}) \lambda_{m}^{FD} \lambda_{m,2}} \right) \]. 

(18)

Proof:

It can be computed that:

\[
B = \Pr \left( \gamma_{SR2,n_{1}} \geq \lambda_{2}^{FD}, \gamma_{BU12,n_{2}} \geq \lambda_{2}^{FD}, \gamma_{BU2,n_{2}} \geq \lambda_{2}^{FD} \right) 
\]

\[
= \Pr \left( \frac{P_{c} a_{2}^{2} |h_{i2}|^{2}}{\alpha P_{c} |h_{i2}|^{2} + N_{0}} \geq \lambda_{2}^{FD} \right) \times \Pr \left( \frac{P_{c} a_{2}^{2} |g_{n,2}|^{2}}{P_{c} a_{2}^{2} |g_{n,2}|^{2} + N_{0}} \geq \lambda_{2}^{FD} \right) 
\]

\[
= B_{1} \times B_{2}. 
\]

We can computed each part as follows:

\[
B_{1} = \Pr \left( \frac{P_{c} a_{2}^{2} |h_{i2}|^{2}}{\alpha P_{c} |h_{i2}|^{2} + N_{0}} \geq \lambda_{2}^{FD} \right) 
\]

\[
= \frac{P_{c} a_{2}^{2} \lambda_{m}^{FD}}{\lambda_{2}^{FD} \alpha P_{c} \lambda_{m}^{FD} + P_{c} a_{2}^{2} \lambda_{m}^{FD}} \exp \left( \frac{\lambda_{2}^{FD} N_{0}}{P_{c} a_{2}^{2} \lambda_{m}^{FD}} \right), 
\]

(20)

and

\[
B_{2} = \Pr \left( \frac{P_{c} a_{2}^{2} |g_{n,2}|^{2}}{P_{c} a_{2}^{2} |g_{n,2}|^{2} + N_{0}} \geq \frac{N_{0} \lambda_{2}^{FD}}{P_{c} a_{2}^{2} |g_{n,2}|^{2} + N_{0} \lambda_{2}^{FD}} \right), \text{if } a_{2}^{2} > \lambda_{2}^{FD} 
\]

\[
= \exp \left( - \frac{\lambda_{2}^{FD}}{\lambda_{m,2}} \right), 
\]

(21)

Then, we have:
\[
OP^{FD}_2 = \prod_{n=1}^{N} \left( 1 - \frac{P_n a^2_n \lambda_{bn}}{\lambda^{FD}_2/P_n + P_n a^2_n \lambda_{bn} + P_n a^2_n \lambda_{bn}} \exp \left( \frac{\lambda^{FD}_2 N_0}{P_n a^2_n \lambda_{bn}} - \theta^{FD} \right) \right),
\]

(22)

where

\[
\lambda^{FD}_2 = 2^{\theta^{FD}} - 1, \quad \theta^{FD} = \max \left( \frac{N_0 \lambda^{FD}_1}{P_1 \left( a^2_1 - a^2_2 \lambda^{FD}_1 \right)} \frac{N_0 \lambda^{FD}_2}{P_2 a^2_2} \right).
\]

Similarly, in HD mode such outage metric for user D2 is formulated by:

\[
OP^{HD}_2 = \prod_{n=1}^{N} \left( 1 - \exp \left( - \frac{\lambda^{HD}_2 N_0}{P_n a^2_n \lambda_{bn}} - \theta^{HD} \right) \right),
\]

(23)

where

\[
\lambda^{HD}_2 = 2^{\theta^{HD}} - 1, \quad \theta^{HD} = \max \left( \frac{N_0 \lambda^{HD}_1}{P_1 \left( a^2_1 - a^2_2 \lambda^{HD}_1 \right)} \frac{N_0 \lambda^{HD}_2}{P_2 a^2_2} \right).
\]

4. SIMULATION RESULT

In this section, we present numerical results to evaluate analytical expressions calculated in previous part. The power allocation coefficients of NOMA is \( a_1 = 0.8 \) for D1. As the observation, Figure 2 and Figure 3 plot the outage probability for proposed NOMA for two far NOMA users described as in figures. In this scenario, relay selection scheme is applied for further improvement of outage behavior. Observing the Figure 2, one can conclude that much power allocated to user D1 results in better outage performance. In addition, both Figure 2 and Figure 3 manifest that RS NOMA can remarkably increase the outage performance as if reasonable selection of FD/HD mode is given. Moreover, when we change transmit power at relay, the gap performance regarding the outage probabilities achieved by RS NOMA can be seen clearly at such high transmit power at relay.

\[ \begin{align*}
\text{Figure 2. Comparison on FD and HD of both far users versus transmit SNR at the BS as varying } & \\
& a_1 (\lambda_{bn}=\lambda_{pn1}=1, \lambda_{pn2}=10, \lambda_{bn}=0.01, P_R = P_T = 20(dB), R_1 = 0.5, R_2 = 1, N_1 = 1, N = 1) \\
\end{align*} \]

\[ \begin{align*}
\text{Figure 3. Outage performance comparison on FD and HD of both users versus transmit power at relay as varying } & \\
& P_U (\lambda_{bn}=\lambda_{pn1}=1, \lambda_{pn2}=10, \lambda_{bn}=0.01, P_R = 20(dB), R_1 = 0.5, R_2 = 1, N_1 = 1, N = 1) \\
\end{align*} \]

In Figure 4, it can be observed that the outage probability varies according to the different values of self-interference channel. The exact outage probability curves of proposed RS NOMA with higher level of SI
will be resulted in worse outage performance. It is observed that the superiority of full duplex function in RS NOMA is no longer apparent with the very large values of SI (i.e. as $\lambda_{in} = 1$ (dB)). Therefore, it is essential to consider the influence of SI when designing practical full duplex antenna in such NOMA systems.

In Figure 5, we evaluate impact of the number of relay at relay selected. Better performance can be observed at higher number of relay selected. We compare the outage behavior of different cases considering the number of relay. It can be observed from Figure 5 that for the proposed scheme, the analytical outage is improved at $N = 2$. In addition, Figure 5 also shows that the proposed scheme can obtained clear outage performance gap as higher SNR raised.

**CONCLUSION**

In this study, we suggested applying relay selection scheme in NOMA scheme to evaluate system outage performance. As most important thing, full-duplex scheme is also studied. The considered NOMA scheme is assessed and compared with the different scenarios in terms of system outage. By achieving outage with appropriate selection of number of relay, it can be observed from the simulation result that the proposed NOMA with improved performance can be applied in real NOMA design. For further research topics, with more than two users we may consider a generalization of relay selection policies and performance investigation for randomly distributed NOMA users.

**REFERENCES**


