Learning Combinations through “Handshake”

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Abstract
This research used design research. It was constructing a learning trajectory through handshake context to help students’ understanding of the Combination concept. This research used PMRI approach. The participants of the research were the students of 10th class SMA Negeri 15 Palembang. For retrospective analysis, the data were collected from a teaching experiment in form of student’s work, field notes, and interviews. The obtained data is a learning trajectory which is consists of 1) students do a role-play scene of the handshake activity; this activity supports students’ understanding that A do handshake with B is the same with B do handshake with A, which it indicates order does not matter in Combination. 2) students utilize their knowledge about factorial forms to analyze the general pattern of the Combination. 3) students solve problems related to the Combination concept to help them think about how to solve the similar problems in the daily life. The results of the study showed that the use of the handshake context can help students to understand of the Combination concept.

Keywords: handshake, combinations, design research, PMRI


INTRODUCTION
Probability is often found in daily life hence it is one of the important topics in mathematics. Finding the likelihood amount of an event needs the probability calculation sciences, one of them is by using filling slots manually. However, in order to know how much the likelihood of an event where objects or data are so many, thus the filling slots method becomes less effective (Gordon, 2006; Purnanto, 2014). According to the problem above, therefore it is needed a more simplified method.
Combinatorial is a branch of mathematics which studies the arrangement of objects without having to enumerate all of the probable arrangements (Munir, 2010; Febriana et al., 2017) and the probability of an event can be calculated fast using the combinations concept since combinations is a part of combinatorial. Based on Gordon (2006), “Combination is a technical term meaning ‘selections’, we use it to refer to the number of different sets of a certain size that can be selected from a larger collection of objects where order does not matter “. Harini (2010) stated that combination is an arranging of objects consists of several elements which do not consider the order. Combinations case is an experiment on objects in form of set H which produces sample space where its sample dots also do not allow repetition of H elements but the order of the H elements on each sample dot is not concerned (Raharjo, 2004).

Research of Putra et al. (2017) showed that students still have difficulties in solving problems related to combinations. Students are confused about when to use the formula of combinations, for the book-based topic presentation has caused them to follow the given problem-solving way, thus they became confused in deciding whether it is combinations or permutations once the question is changed. Based on (Duffin & Simson 2000; Kesumawati 2008), developing few consequences of a concept may imply that student understanding of a concept impacts the ability to solve every problem correctly. In learning mathematics, students have to understand the mathematical concept first in order to be able to solve the given questions and to apply the learning in the real world (Murizal et al., 2012). This is parallel to the research of Sukoriyanto et al. (2016) which stated that student error is high in understanding questions related to combinations. This is also supported by the research of Sina (2011) which said that students of the 11th class in senior high school perceive probability lesson containing combinations as a hard topic for there are too many calculations, formulas which have to be memorized and that they have to abstract/imagine a theory in daily life. In other words, the conceptual understanding of mathematics significantly affects student ability in solving problems such as problems related to mathematics. For that reason, a teacher creativity in designing inside-class learning is needed so the students will truly understand the concept of the learning.

Based on the problem above, a proper approach is needed, i.e. Pendidikan Matematika Realistik Indonesia (Indonesian Realistic Mathematics Education). Since 2001, Pendidikan Matematika Realistik Indonesia has been commonly used in the effort of improving student interest, attitude and learning result (Zulkardi, 2009). Pendidikan Matematika Realistik Indonesia is a learning approach adapted from Freudenthal’s thinking known as Realistic Mathematics Education and has been developed in Indonesia. Pendidikan Matematika Realistik Indonesia can be used by mathematics teachers to develop student skills in thinking, reasoning, communicating and solving problems both in the lesson and daily life (Zulkardi, 2002). Pendidikan Matematika Realistik Indonesia has three main principles which are Guided reinvention, Didactical phenomenology and Self-developed model (Gravemeijer, 1994).

Several previous types of research conducted e.g. Diana et al. (2016) stated that Pendidikan Matematika Realistik Indonesia can help students in understanding mathematical learning concept. Other researchers also said that the usage of Pendidikan Matematika Realistik Indonesia approach has a significant role for it can be used as a starting point in learning and support student skill of solving the problem in daily life (Yulianita et al. 2016). This is also parallel to the statement of Putri (2007) which assumed that student low achievement at school is caused by learning topics which are not interesting enough due to the inadequate examples applied in daily life and the learning method which is more focused on the teacher.
Pendidikan Matematika Realistik Indonesia can be interesting and useful for students. Since through Pendidikan Matematika Realistik Indonesia, they can use context as a starting point in learning. In this research, we used handshake context as the starting point in learning. According to Kamus Besar Bahasa Indonesia, the handshake is shaking each other’s hand or giving greeting by shaking hands. Based on Gordon (2006) combinations is a technical term which means “selection” and Ross (2010) said combinations can start from determining the group amount of different r objects which can be formed from the total of n-objects as if determining or selecting two committee members from four people and so on. The handshake context is chosen for it can interpret the combinations where the handshake itself is r object and all people doing the handshake is n object. This handshake activity will be done by role-playing for Wroughton and Nolan (2012) said that combinations can be solved using games. This is parallel to the research of Wijaya et al. (2011); Prahmna et al., (2012); Nursyahidah et al., (2013) which said that the concept of games can be a real experience for student hence it can be used as a starting point/opening activity/set of activities for learning process.

According to the explanation above, the question of this research is "how does learning trajectory of combinations use handshake context in helping students to understand the concept of combination".

RESEARCH METHOD

The research method is design research which produces a learning trajectory in the learning of the combination concept by using handshake context. Design research aims to develop a local instructional theory which is based on the existing theory (theory-driven) and empirical experiment (empirically based) through cooperation between the researcher and the teacher to improve the relevancy of this research with educational policy and practice (Gravemeijer & Van Eerde, 2009). According to (Gravemeijer, 2004), design research has three phases, which are: preparing for the experiment, the design experiment, and the retrospective analysis. In the phase of preparing for the experiment, researcher reviewed all literature which would be used in the research to produce learning trajectory or Hypothetical Learning Trajectory. Hypothetical Learning Trajectory is designed with the containing of anticipation for possible occurrences. Before designing, the researcher determined to learn purpose or aimed purpose and initial point of learning.

The phase of design experiment is divided into two cycles, which are the pilot experiment and the teaching experiment. Pilot experiment or initial experiment of this research is known as a bridge between a phase of initial design and phase of teaching experiment. The purpose of this research is to test initial Hypothetical Learning Trajectory. The main objective of this phase is to collect data for supporting correspondence with the initial Hypothetical Learning Trajectory. In this phase, Hypothetical Learning Trajectory which has been designed was tested to students in a non-research-subject class. Next is teaching experiment, which in its cycle has data collection activity to answer the research question.

The phase of teaching experiment is the implementation of Hypothetical Learning Trajectory which has been improved and implemented in a real class. The purpose is to explore student strategy and thinking in a real learning, as data which will be used to answer the research question (Gravemeijer, 2004; Bustang et al., 2013).

The last phase is retrospective analysis. All of the obtained data analyzed retrospectively were used to plan a learning activity or to develop a design on the next learning activity, with a purpose of developing Local Instructional Theory. In this phase,
Hypothetical Learning Trajectory is compared to student real learning process to answer the research formulated problem. The research phases can be seen in Figure 1.

![Design Research Phases](image)

Figure 1. Design Research Phases (Gravemeijer, 2004)

The cycle of teaching experiment is the focus of this research discussion. The teaching experiment was established in class 10th Science 3 of SMA (Senior High School) 15 Palembang which consists of 36 students.

**RESULTS AND DISCUSSION**

Design research has 3 phases which are preparing for the experiment, the design experiment, and the retrospective analysis. In a phase of the design experiment, there are two cycles: the first cycle is a pilot experiment and the second cycle is teaching experiment. The cycle of teaching experiment next will be the focus of this research discussion.

In the first cycle, after implementing pilot experiment, a revision was established according to the findings, results of observation and analysis of student's answers on the implemented pilot experiment. The revision was done in order to get an optimal result on the teaching experiment. The researcher discussed with a model teacher about improvements done for the teaching experiment. The revision was performed on question number 3 activity 2. On the question 3, researcher improved its language aspect. In that question, there was "jumlah" word which caused the students confused to fill their answers for the sentence was unclear. Hence, the researcher corroborated the sentence by adding "pada kolom 'jumlah' (on 'amount' column)". Figure 2 and Figure 3 is the improvement for the question number 3.

![Question 3 Prior to Revision](image)

Figure 2. Question 3 Prior to Revision
After performing a revision on the question number 3, a revision was done on question number 4 activity 2. On the question 4, the researcher also improved its language aspect. There was a vague sentence on that question so the students were confused in filling their answers. Thus, researcher corroborated the sentence by adding "jawaban pada kolom jumlah" (answer on amount column). Figure 4 and Figure 5 is the figure of the improvement for the question number 4.

Table 1 shows the final Hypothetical Learning Trajectory which has been designed for further usage on teaching experiment.
Table 1. Overview of the HLT

<table>
<thead>
<tr>
<th>Sequence of activities</th>
<th>Goals</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handshake roleplay of the two main teams of takraw after their match ended.</td>
<td>Students are able to do handshake roleplay of the two main teams of takraw after their match ended.</td>
<td>In this activity, students have to determine all players of the two main teams (one team consists of 3 players). After that, they perform the handshake roleplay scene after the takraw matched is finished. Then, they have to determine how often a handshake occurs on the given problem. In this problem, they have to understand that A shakes hands with B is the same with B shakes hands with A. This occurrence is the fundamental concept of combinations, where the order is not a concern.</td>
</tr>
<tr>
<td>Analyzing the general pattern of combination based on student understanding about factorial form</td>
<td>Students are able to analyze the general pattern of combination based on their understanding about factorial form obtained from the handshake activity.</td>
<td>Students are asked to count how often a handshake may occur between 2, 3, 4 and 5 people by modeling them using symbols they like such as a, b, etc. After that, they have to determine what will happen if the order of the handshake activity is concerned, then transform it into the nearest multiplication form. Next, they have to transform it again into factorial form and then with their understanding on factorial, they analyze the general pattern of the combination itself.</td>
</tr>
<tr>
<td>Solving problems related to combinations</td>
<td>Students are able to solve problems related to combinations.</td>
<td>Students are given two problems related to combinations. In the first problem, students have to use their knowledge about how to determine r and n elements also the general pattern of the combination. Next, in the second problem, similar to the first one but students also have to use their knowledge about multiplication rules.</td>
</tr>
</tbody>
</table>

In the implementation phase of teaching experiment, researcher acted as the observer by examining students’ strategy in solving the given problem. Hypothetical Learning Trajectory used here was the revised result of the pilot experiment.

The teaching experiment was performed in class 10th Science 3 SMA Negeri 15 Palembang with 36 students. Mrs. Dra. Yulianita, M.Pd., the homeroom teacher acted as a model teacher in this phase. In the teaching implementation, teacher divides the students into six groups of six students (group 1-6). The students were grouped based on homogenous ability among them and heterogeneous ability in each group, where every group consists of students with high, middle and low ability.

Students started the first activity by doing a handshake roleplay scene off the two teams of takraw after their match ended. This sport is originated from the Malay Sultanate era (634-713) and is known as sepak raga in Malay. Takraw is played by two
teams with three main players each. This game uses a net and numeric rule. This game is called *sipa* in Philippines, *chinlone* in Burma, *maradong* in Laos, and *takraw* in Thailand.

Every group conducted the discussion in order to determine the number of people in the two main teams of *takraw*. After that, students did the handshake scene among players where student A did handshake with student B and so on. Thus, students were asked to determine how often the handshake will occur in the given problem. Figure 6 is a figure of students doing the role-play handshake scene.

![Figure 6. Students Role-played Handshake](image)

When doing the activity, students were a bit confused in answering how often the handshake occurred in the given problem. Students initially answered 16 times of handshake occurred on both of the main teams then the teacher guided them in solving the problem. Transcript 1 is the conversation transcript.

**Transcript 1**

*Teacher*: Write down the probability of whoever doing the handshake? Try it!

*Student*: One, two, three... sixteen. (role-playing the handshake while counting it one by one).

*Teacher*: Okay, now handshake of Rusdi with Tri. Between the handshake of Rusdi with Tri and the handshake of Tri with Rusdi, are those handshakes the same?

*Student*: Yes, they are, mam.

*Teacher*: How often will the handshake occur?

*Student*: Once.

*Teacher*: Hmm? Let’s try it again, then.

*Student*: One, two, three... fifteen. (role-playing the handshake while counting it one by one).

*Teacher*: Fifteen or sixteen?

*Student*: Fifteen.

In transcript 1 it is shown that question like “Between the handshake of Rusdi with Tri and the handshake of Tri with Rusdi, are those handshakes the same?” was very helpful for students to understand the basic combinations concept itself. Since in combinations AB = BA, which means that its order is not concerned.
In the second activity, students were hoped to be able to analyze the general pattern of combination from a handshake problem. They were asked to determine the number of people that would do handshake by modeling them with symbols and calculate the possible results of the handshake experiment. Transcript 2 is the conversation transcript.

**Transcript 2**

*Teacher*: \( \{a,b\} \) what are these?
*Student*: \( \{a,b\} \) these means 2 people, thus there are 2 people assumed to be \( a \) and \( b \)
*Teacher*: ooh I see, after that?
*Student*: the experiment way is 2
*Teacher*: why 2?
*Student*: because we are shaking hands, handshake can only be done by 2 people, and impossible if only 1 person, therefore the experiment way is 2, the possible result of 2 people is 1 result
*Teacher*: what 1?
*Student*: 1 time of handshake

Based on the transcript 2, it can be seen that students were able to explain the origins of their answers by assuming/modeling the available people with symbols of \( a \), \( b \), etc. After that, they were asked to calculate how often the handshake may occur from 2, 3, 4 and 5 people. They were able to determine the possibilities that may happen in an experiment related to combinations, which in the given problem is about shaking hands.

According to their solved result in the given activity: if there are 2 people available, thus the handshake that may occur in 1 time of handshake; if there are 3 people available, thus the handshake that may occur in 3 times of handshake; if there are 4 people available, thus the handshake that may occur is 6 times of handshake; and if there are 5 people available, thus the handshake that may occur is 10 times of handshake. Student's answers can be seen in Figure 7.

![Figure 7. Student’s Answers in Determining the Handshake Occurred](image)

Next, in this activity students were asked to determine the amount on the given handshake problem if the order was concerned and transform it into the nearest multiplication form, e.g. for 12 arrangement forms if transformed into its nearest
multiplication form becomes \(3 \times 4\) or \(4 \times 3\). After the students were able to find the nearest multiplication form, they were asked to transform it into factorial form, e.g. \(4 \times 3\) transformed into \(4! / 2!\). Based on student's answers showed that they were able to solve the problem on the given activity. Their answers can be seen in Figure 8.

![Figure 8. Student’s Answers in Determining the Handshake Occurred, then Transformed It into the Nearest Multiplication and Factorial Form](image1)

After that, students analyzed the general pattern of combination based on the previous activity and student understanding of the factorial concept. Their answer in analyzing the general pattern of the combination is shown in Figure 9.

![Figure 9. Student’s Answers in Analyzing the General Pattern of Combination](image2)

According to student’s answers on Figure 9, it is shown that students were able to analyze the general pattern of combination based on the previous activity and their understanding about factorial. In analyzing the general pattern, they understood that after obtaining the factorial form, they had to divide it with \(r\) factorial which in the given problem was \(2!\). Transcript 3 is the conversation transcript between teacher and student.
Transcript 3

Teacher: why divided by 2!?
Student: because, from this r
Teacher: why divided by 2!?
Student: thus the result will be the same with the column of total amount on question number 1
Teacher: how many?
Student: the result is 1
Teacher: this \( \frac{5!}{3!} \), how did you get the "3!"?
Student: 5-2, the amount of people is subtracted with its experiment way, after looking at this one, this is divided with its experiment way (pointing at the answer)
Teacher: also this one?
Student: yes, same for all, in order to get result like the column of amount on the question number 1
Teacher: then what is the conclusion?
Student: the conclusion is \( \frac{n!}{(n-r)!r!} \)

On transcript 3, it is shown that students understand that in order to obtain combination result (where in this problem is located on the column of amount on question 1 which can be seen in Figure 7) which they found in the previous problem, they have to divide all of the factorial forms they obtained with r factorial thus they found that the general pattern of combination is \( \frac{n!}{(n-r)!r!} \). The student's answers can be seen in the previous Figure 9.

The third activity is where students solved problems related to combinations. In this problem, they were given two problems. First, they had to use their knowledge on how to determine r and n elements and the general pattern of the combination. Second, similar to the first one but they also had to use their knowledge about multiplication rules. The first problem was as follows, "An organization in Indonesia has 15 people that are experts in music. One day, there will an international competition which can only be participated by 4 representatives of every nation. How many ways of Indonesian representatives that may be chosen for the competition?". The student's answers to the first problem can be seen in Figure 10.

According to student's answers on Figure 10, it is shown that students were able to determine n object and r object of the given problem where the n object was 15 experts and the r object was the selection of 4 representatives for the international competition. In addition, students were also able to use the general pattern of combination in solving the given problem, which the final result was 1365.
Next, the second problem was as follows, "An organization has several experts, which are 4 machine (Mesin) experts and 4 electronic (Elektronika) experts, that one day will be positioned as judges. Make judges arrangements consist of 1 machine experts and 2 electronic experts!". Student’s answers can be seen in Figure 11.

![Figure 11. Student’s Answers in Activity 3 Problem 2](image1)

According to the student’s answers on Figure 11, it is shown that students were able to determine n object and r object, where for the electronic experts: n object was 4 electronic experts and r object was the selection of 2 electronic experts; while for the machine experts: n object was 4 machine experts and r object was the selection of 1 machine expert. In addition, students were also able to use their knowledge about the general pattern of combination and multiplication rules in solving the given problem where the final result is 24 kinds of judge’s arrangement.

Based on student’s answers showed that students were able to understand the combinations concept and at the end of the activity they could analyze the general pattern of combination—where that general pattern is a formula of the combination itself—then utilized it to solve problems related to combinations with guidance and help as needed from the model teacher. Vygotsky; Sani (2013) stated that scaffolding term is a knowledge construction process which is done together with help as needed. Discussion between a student with student, the teacher with student in solving problems which are realistic, the teacher helps/explains as needed if the students experience any difficulty in understanding the problem and making the conclusion. This shows that the Pendidikan Matematika Realistik Indonesia approach is parallel to the theory of social constructivism.

In the handshake role-playing activity done by students, it is shown that handshake can help students in understanding the combinations concept. This is parallel to the opinion of Ross (2010) which said that combinations can starts form determining the amount of different r object groups that may be formed from the total of n object, where in this research the n object is all players in two main teams of takraw and the r object is the experiment way which is handshake. With the handshake context which was done by role-playing, students were able to solve the given problems easier. In a group discussion, students did the handshake role-playing activity enthusiastically. This shows that they were very interested in a program which uses activity perceived to be new to them.

According to retrospective analysis on activity 1, 2 and 3 in cycle learning 2 (teaching experiment), the learning was in accordance with the Hypothetical Learning Trajectory arranged and it can be concluded that students have understood the combination concept through role-playing the handshake. This is in line with a statement of Wroughton and Nolan (2012) which said that combinations can be solved
using games, and that roleplays can be used to help students understand mathematics or science aspects deeper. Furthermore, these can help students to be more attracted and involved to not only learn the topics but also to integrate their knowledge in action by solving problems and seeking for creative solutions (Blatner, 2002; Sina, 2001).

In this learning process, this study also has reflected the three principles of *Pendidikan Matematika Realistik Indonesia*. Just like the statement of Zulkardi & Putri (2010) that the three principles of *Pendidikan Matematika Realistik Indonesia* are guided reinvention and progressive mathematizing, didactical phenomenology, and self-developed models. The first principle is guided reinvention, students in the learning process of combinations found answers through teacher’s guidance and usage of the handshake context. Students have found elements e.g. n object and r object of the given problems.

The second principle is didactical phenomenology, the given learning situation which was derived from phenomenon or incidence occurring in daily life and able to be understood by students is very important in learning implementation in order to make students learning of mathematics concept easier. In this case, the phenomenon occurring in daily social life was used in the learning of combination topic in order to make the students understand the combination concept easily. Hence a role-playing activity with handshake context was used in this research.

The third principle is self-developed models where the model which would be developed may act as a bridge for students in order to understand the knowledge they get, starting from contextual problem in real situation to problem in abstract situation, the model development acted as the bridge for students from the real situation to the mathematical formal situation. This can be seen when students determined the number of people doing the handshake, the handshake occurred, transformed them into symbol forms, transformed these forms into the nearest multiplication forms, transformed the nearest multiplication forms into factorial concept forms and in the end, they were able to find the general pattern of combination as the result of student understanding of the combination concept.

Other than the three principles of *Pendidikan Matematika Realistik Indonesia*, the designed learning activity also reflected the five characteristics of *Pendidikan Matematika Realistik Indonesia*. The first characteristic is the use of contexts for phenomenologist exploration where the learning activity was started with the contextual problem often met by students as an experience-based problem. Every activity in the learning used problem with the handshake context. Based on the given context, it is shown that most students have understood and known the handshake context thus it can be integrated into learning.

The second characteristic is the use of models for mathematical concepts construction aiming to connect student understandings in abstract form into the real form which is known as the transition of informal form into formal form (Gravemeijer, 1994). According to Gravemeijer (1994), there are levels of Realistic Mathematics Education (RME) which are situational level, referential level, general level and formal level. The handshake problem which can be imagined by students is a model of combination learning of the referential level. By modeling it into symbols like a, b and others that the students prefer to bridge their understanding from the abstract into real can help them in learning combination.

The third characteristic is the use of students’ creation and contribution. This characteristic is shown in the combination learning process of the given set of
activities. The teacher gave appreciation to students’ contribution in the learning process, whether it was a group activity or individual activity. Students were given the freedom to express and answer questions using their own strategy. In addition, the students that succeed in finding problem-solving strategy of the combination were able to guide other students in their group while discussing. This thing occurred in each activity, 1-3. Other than that, the teacher acted as a facilitator in each activity and was not too dominating the learning so the students could be creative according to their understanding.

Next, the fourth characteristic is about students’ activity and interactivity in the learning process. The interactivity between students and teacher or among students appeared in every activity, whether it was a discussion activity or individual activity. The last characteristic of this learning is about the intertwining mathematics concepts, aspects, and units. The design of this learning is not off of its intertwining with other topics.

CONCLUSION

Based on the research results and discussion, it can be concluded that the learning trajectory obtained consists of 3 activities as follows: Activity 1, students do handshake role-playing scene of two main teams in takraw after finishing a match to understand the combinations concept. Through this activity, students understand that combinations are the selection of objects which consists of several elements without considering the order. Activity 2, students analyze the general pattern of the combination through their understanding and knowledge about factorial form. Activity 3, students solve problems related to combinations, they can solve these problems by utilizing their knowledge about the general pattern of the combination and rules of multiplication. Other than, the results showed that the usage of the handshake role-playing game context can help students in understanding the combinations concept.

REFERENCES


