**International Journal on Emerging Mathematics Education (IJEME)**
Vol. 6, No. 1, March 2022, pp. 63-76
P-ISSN: 2549-4996, E-ISSN: 2548-5806, DOI: http://dx.doi.org/10.12928/ijeme.v6i1.17336

# Characteristics of Different Strategies in Problems Solving of Linear Pattern

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**Abstract**

Generalization is the core of mathematical activities that are important to teach to the students. One of the material generalizations emphasized at the junior high school level in Indonesia is linear pattern generalization. One strategy that students often use in generalizing linear patterns is a different strategy. However, many students do not know the usefulness of the difference in making general formulas, so that they are confused in a recursive relationship. The solution to this problem is by analyzing the results of student work that has succeeded in generalizing linear patterns using different strategies. For this reason, this study aims to get a description of the characteristics of the different strategies of students who have succeeded in generalizing linear patterns. The approach that fits this research is a case study approach to six 8$^{th}$ grade junior high school students who successfully solved generalizing linear patterns using a different strategy. The results showed that there were six characteristics of different strategies used to generalize linear patterns, namely: (1) using the difference to be substituted into the nth term formula of an arithmetic sequence, (2) using the difference to substitute into linear pattern formula, (3) using difference as a multiple, (4) using the difference as a jump number, (5) using the difference to place in a different column, and (6) using the difference to determine the formula for generalizing linear patterns directly.

**Keywords**: Characteristics of different strategies, Generalization strategy, Linear pattern, Quantitative relationships

***How to Cite***: Setiawan, Y. E. (2022). Characteristics of different strategies in problems solving of linear pattern. *International Journal on Emerging Mathematics Education, 6*(1), 63-76. http://dx.doi.org/10.12928/ijeme.v6i1.17336

## INTRODUCTION

One of the generalization materials taught in schools is pattern generalization (Ellis, 2011b; Kemendikbud, 2016; NCTM, 2000). Patterns generally consist of repeating patterns and growing patterns (Mulligan, 2010; Mulligan & Mitchelmore, 2009). Repeating patterns are patterns that have a cyclic structure produced by repeated applications of smaller parts of the pattern, whereas growing patterns are patterns that grow over time and develop from step to step (Hourigan & Leavy, 2015; Setiawan, 2020c; Setiawan, Purwanto, Parta, & Sisworo, 2020). In this study, researchers used a growing pattern. Growing patterns are likely to form linear and quadratic patterns (Stacey, 1989) or exponential patterns (Ellis, 2007b, 2007a, 2011b, 2011a). A linear pattern is a growing pattern that if the first difference of the terms is constant, while the quadratic pattern is a growing pattern, the second difference of the terms is constant (Liljedahl, 2004). By following the n$^{th}$ term, the linear pattern is expressed in form $U_n = an + b$ with $a \neq 0$, and the quadratic pattern is expressed in form $U_n = an^2 + bn + c$ and $a \neq 0$. This research focuses on linear patterns (Liljedahl, 2010; Setiawan, 2020c; Setiawan et al., 2020). The focus of research is on linear patterns because learning at the junior high school in Indonesia emphasizes linear pattern generalization (Setiawan, 2020f; Setiawan et al., 2020).

*Received July 14, 2020; Revised March 30, 2021; Accepted January 4, 2022*

Research on the generalization of linear patterns has been carried out. The study results (Becker & Rivera, 2005) show that 9th-grade junior high school students can use various generalization strategies, namely visual strategies, numerical strategies, and pragmatic strategies. The visual strategy is a strategy of generalizing linear patterns using images. A numeric strategy is a strategy of generalizing linear patterns using numbers, whereas pragmatic strategies combine visual strategies and numeric strategies. One of the numerical strategies in generalizing linear patterns is a different strategy (Becker & Rivera, 2005; Setiawan et al., 2020; Stacey, 1989), which refers to pattern generalization strategies using differences (Setiawan et al., 2020).

However, there are still many students who cannot use different strategies to generalize linear patterns. Study results by Becker & Rivera (2005) find that some students cannot generalize linear patterns using this different strategy. Another study results (Hourigan & Leavy, 2015) also show that if the students focus on numerical data, i.e., differences in geometric growth patterns, they will misunderstand in a recursive strategy. Furthermore, this recursive strategy cannot be used as a pattern generalization strategy because it does not find general rules (Hourigan & Leavy, 2015). The results of the research of (Setiawan et al., 2020) also show that students who did not know the usefulness of the difference in finding the $n^{th}$ term formula of a linear pattern would confuse into a recursive strategy; a strategy that wrote down all the terms (Setiawan, 2020f). From previous studies, it is clear that students still experience problems using different strategies. For example, the students do not know the usefulness of the difference in generalizing linear patterns.

It is crucial to analyze the students who have successfully used different strategies to generalize linear patterns to overcome the unresolved students' problems in previous studies; which students' problem is not knowing the use of differences between terms in generalizing linear patterns. This analysis of student work outcomes aims to find the characteristics of different strategies used in generalizing linear patterns. So, this study aims to characterize the different strategies used by students in generalizing linear patterns. The main benefit of this study's results is to produce a theory about the characteristics of different strategies in generalizing linear patterns. This study's results also help teachers provide generalized linear patterns using various characteristics of different strategies.

## RESEARCH METHOD

This type of research is a descriptive qualitative study using a case study approach to describe different strategies' characteristics in generalizing linear patterns briefly. A case study is an approach that uses several systematic procedures that are useful for developing theory, where cases are studied in-depth. The results are used to formulate new theories (Creswell, 2012). The new theory generated through this research is the various characteristics of different strategies for generalizing linear patterns.

This research was conducted in two public junior high schools in Lumajang district that were accredited A. This research was conducted in the odd semester of the 2019-2020 school year. The subjects of this study were students of grade $8^{th}$ who were divided into two classes. The selection of this subject is according to the fact that grade eight students have studied number pattern material, including linear patterns. Before starting the research, the researcher asked the principal for permission to research the school. After obtaining permission from the principal, researchers immediately conducted research. Ethically, researchers kept the subjects' identity in this study a secret that aims to protect the subject.

The subjects in this study consisted of 6 students. The research subject's determination was carried out by asking 40 grade 8th junior high school students in two public schools in Lumajang to solve generalizing linear patterns (see Figure 1). The work of 40 students was categorized based on the generalization strategy used. From 40 students, 29 students solved the linear pattern problem using different strategies. From 29 students, the characteristics of different strategies in generalizing linear patterns are available in Table 1. From Table 1, one student participated as the research subject of each of the different strategic characteristics.

**Table 1.** Characteristics of Different Strategies

| Characteristics of different strategies | Description | Number students | % |
|---|---|---|---|
| Substituting the difference to the nth term formula arithmetic sequence. | The strategy that uses the difference to substitute into the nth term formula from the arithmetic sequence, namely $U_n = a + (n-1)b$ | 11 | 37,93% |
| Substituting the difference to the linear pattern formula. | The strategy that uses the difference to substitute to the linear pattern formula is $U_n = bn + c$ | 1 | 3,45% |
| Using difference as a jump number. | The strategy uses difference as a jump number. | 3 | 10,34% |
| Using differences as multiples | The strategy using differences as multiples is $bn$. | 2 | 6,90% |
| Placing differences in different columns | Strategies that use differences to substitute into different columns, from the formula $U_n = (n \times ...) + c$ | 1 | 3,45% |
| Use the difference to determine formulas directly. | Strategies that use the difference to directly determine generalization formulas from linear patterns | 11 | 37,93% |
| Total students | | 29 | 100% |

The data collected in this study consisted of the subject's work and transcripts of interview results. The procedure for collecting data from the subject's work was carried out by following the research subject's procedure. The data collection procedure in the form of a transcript of the interview results was carried out using two steps. The first step was to conduct interviews with research subjects. The interviews were conducted in a semi-structured manner and last about 10 to 15 minutes. When conducting the interview, the researcher recorded the interview activity using the audio on the cellphone. The second step was to transcribe the interview results so that data was obtained from a transcript of the interview results with the subject. The interview results were carried out by writing down word for word to make it easy to analyze the interview data (Creswell, 2012). Thus, the data was obtained from the work and transcript data from interviews with research subjects.

This research instrument is the problem of generalizing linear patterns, which available in Figure 1. The terms of generalizing linear patterns using number representations. The reason researchers use numbers because in solving the problem of number patterns, a person tends to use different strategies (Setiawan et al., 2020). So, through generalizing this number pattern, a person's characteristics in generalizing linear patterns using different strategies can be identified.

The sequence of numbers is known as follows

6, 10, 14, 18, ...

Determine:
a. Numbers in 5th to 10th terms! Write your way!
b. The general formula for determining numbers in nth terms! write your way!
c. The number in the 57th term! Write your way!

**Figure 1.** The Problem of Generalizing Linear Patterns

Data analysis in the form of student work aims to determine the characteristics of students' different strategies. This difference strategy refers to a generalization strategy that focuses on using differences between terms in number patterns (Becker & Rivera, 2005; Bishop, 2000; Lannin, Barker, & Townsend, 2006; Rivera, 2015; Setiawan, 2020b, 2020f). Therefore, the analysis of student work results was carried out by looking at students' different uses in solving linear generalization problems. These student work analysis results are different generalization characteristics in solving linear pattern problems, as shown in Table 1. The analysis of a transcript from interviews with subjects aims to determine students' stages using different strategies. The researchers themselves analyzed the interview transcripts by coding the words that appeared as themes in the interview transcripts that represented the research findings. This study's findings are in the form of characteristics and steps of different strategies used to generalize linear patterns.

Furthermore, this study's findings were validated by asking questions back to the subjects through interviews, i.e., do they agree with these findings? Moreover, ask them to recheck the accuracy of this study's findings (Creswell, 2012). The six subjects confirmed that these findings were accurate and represent the characteristics and steps of the six subjects' different strategies. Although the interviews in this study were only conducted once, it seemed that the data from the interviews and analyses provide a solid basis for this study's findings. Although this study has a limited number of research subjects, this study's findings provide a theoretical contribution to provide alternative solutions when students have difficulty generalizing linear patterns using different strategies.

## RESULTS AND DISCUSSION

Based on the work analysis results of the six subjects of this study, there are six characteristics of different strategies used in generalizing linear patterns, namely: (1) using the difference to substitute into the nth term formula of arithmetic sequence; (2) using the difference to substitute into a linear pattern formula; (3) using the difference as a jump number; (4) using differences as multiples; (5) using differences to be placed in different columns; and (6) using differences to determine the formula for generalizing linear patterns directly. Each characteristic of these different strategies is explained as follows.

### Substituting the difference to the $n^{th}$ Arithmetic Formula

The first subject (S1) solved the linear pattern problem by using the difference to be substituted into the nth term formula of the arithmetic sequence. This strategy can also be called the strategy of remembering or using arithmetic sequence formulas. The results of the work of this first subject can be seen in Figure 2. Figure 2 proves that the subject found the $n^{th}$ term formula of the linear pattern problem is done by substituting the first term ( $a$ ) and the difference ( $d$ )of the linear pattern terms to the general formula of the n^th^ term Arithmetic sequence namely $U_n = a + (n-1)d$ . By doing the calculations, the subjects obtain the linear pattern generalization in the n^th^ term formula $U_n = 4n + 2$ .

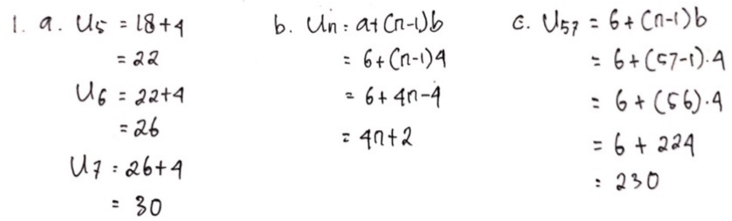


**Figure 2.** First Subject Work Results

The interview excerpt reveals the steps of the subject in using the different strategies as follows.

| | | |
|---|---|---|
| *Researcher* | : | *What do you determine the difference for?* |
| *S1* | : | *To be substituted into the nth term formula of arithmetic sequences.* |
| *Researcher* | : | *How do you get the generalization formula* $U_n = 4n + 2$ *!* |
| *S1* | : | *I substitute the first term and the difference in the nth term formula of the arithmetic sequence. Then I counted, so I got the result* $U_n = 4n + 2$ *.* |
| *Researcher* | : | *How did you know that the patterns form arithmetic sequences!* |
| *S1* | : | *Because they have the same difference.* |

From the interview results, it is clear that the steps of the subject in using the difference to be substituted into the formula of the n^th^ term of an arithmetic sequence, namely: (1) the subject looks for the first term and is different from the arithmetic sequence, (2) the subject substitutes the first term and the difference to the term formula the nth arithmetic sequence, namely $U_n = a + (n-1)d$ , where $a$ is the first term and $d$ is different, (3) the subject does the calculation so that the results of the linear pattern generalization are obtained, namely $U_n = 4n + 2$ .

**Substituting the Difference into Linear Pattern Formulas**

The second subject (S2) solves the linear patterns by substituting the difference into the linear pattern formula. This strategy is also called the remembering strategy or using a linear pattern formula. Figure 3 shows the results of the work of this second subject as follows.

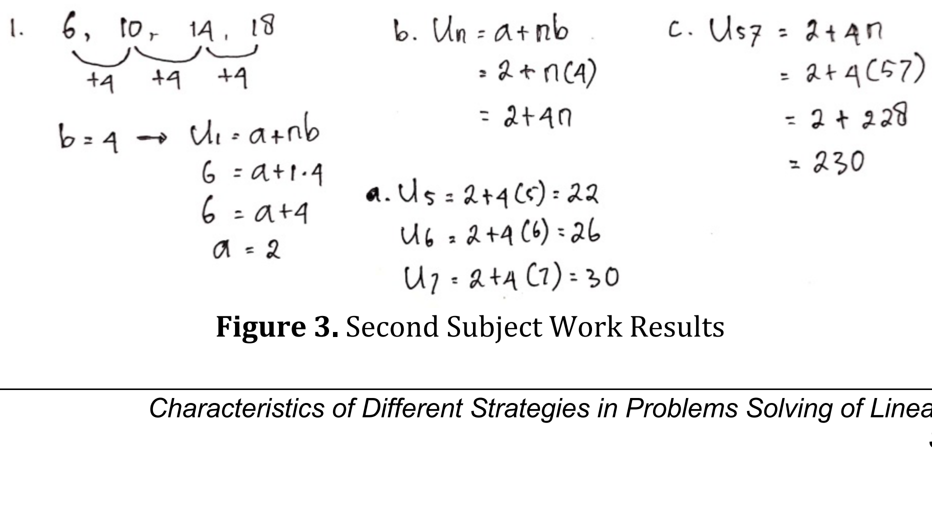


**Figure 3.** Second Subject Work Results

From Figure 3, it is clear that the subject is looking for differences to substitute into a linear pattern formula written by the subject, namely $U_n = a + nd$ (where $d$ is different from terms and $a$ is a constant). After the difference is substituted into a linear pattern formula, the formula $U_n = a + 4n$ is obtained. Subject searching for the value of $a$ using the first term is 6. As a result, the $a$ value is 2. The subject substitutes the value of different and the value of $a$ upon the linear pattern formula, then the result of the generalization of the linear pattern is $U_n = 2 + 4n$. The following interview excerpt reveals the subject's steps in using the different strategies.

| | | |
|---|---|---|
| *Researcher* | : | *What do you determine the difference for?* |
| *S2* | : | *To be included in the linear pattern formula.* |
| *Researcher* | : | *How do you get the generalization formula* $U_n = 2 + 4n$ *?* |
| *S2* | : | *By substituting the difference, then I use the first term to find the value of a from the linear pattern. After the value of a is known, then I substitute it for the linear pattern formula.* |

The interview results reveal that the steps of the subject using the difference to substitute into the linear pattern formula are: (1) the subject looks for a difference from the linear pattern; (2) the subject substitutes the difference to the general formula of the linear pattern, namely $U_n = dn + c$ where $d$ is the difference and $c$ is a constant, (3) the subject determines the value of $c$ using the first term, and (4) the subject substitutes the difference ($d$) and the constant value ($c$) to the linear pattern formula, to obtain the results of the generalization of the linear pattern is $U_n = 2 + 4n$.

**Using Difference as a Jump Number**

The third subject (S3) solves generalizing linear patterns using difference as a jump number. When using a difference as a jump number, it is essential to know the jump number concept. Suppose the problem of generalizing linear patterns has a difference of 4; this means that the jump number is as far as 4, the jump is twice as far as 8, the jump number is three times as far as 12, and the jump number, as far as $n$ times, is $4n$. Figure 4 shows the results of the work of this third subject.

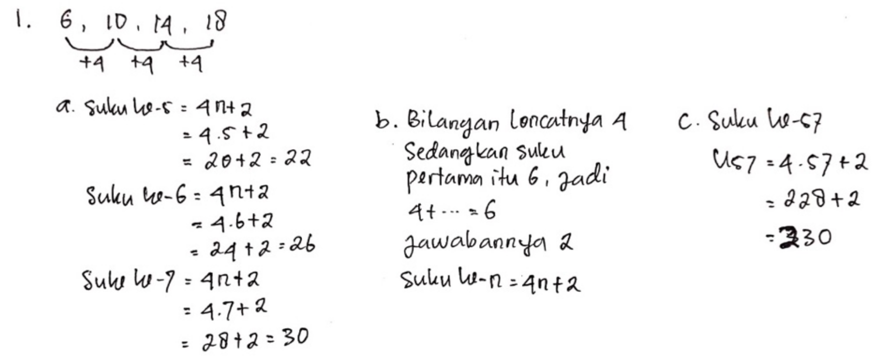


**Figure 4.** Third Subject Work Results

Figure 4 shows that the subject uses difference as a jump number—first, the subject of searching for difference, which is 4. Second, the subject uses the difference as a jump number: a jump number 4 written $4n$. Third, the subject connects the number 4 with the first term using the addition operation (i.e., 4 + ... = 6), the answer is

2. Fourth, the subject writes the formula obtained from the jump number and the jump number relationship with the first term, namely $U_n = 4n + 2$. The interview snippet with the following subject determines the steps for using difference as a jump number.

| | | |
|---|---|---|
| *Researcher* | : | *What do you determine the difference for?* |
| *S3* | : | *To determine the next term.* |
| *Researcher* | : | *How did you get this Generalization formula (the researcher points to* $U_n = 4n + 2$ *)!* |
| *S3* | : | *First, I specify the difference. Then I saw the difference as a jump number 4. While the first term was 6, four-plus how many were the results of 6, which was added 2. So I wrote the generalization formula with the jump number 4 then added 2 packs, namely* $U_n = 4n + 2$. |

From interviews with subjects that use the difference as a jump number, indeed, the steps that the subject used was the difference as a jump number, namely: (1) the subject looked for differences from linear patterns, (2) subjects wrote differences as jump numbers, mathematically written $dn$, where $d$ is different, (3) the subject looked for the relationship of the jump number (i.e., difference) with the first term, namely $U_1 - d = c$, and (4) the subject wrote a linear generalization formula which is a combination of the jump number with the constant $c$ so that the obtained results of the generalization of the linear pattern is $U_n = 4n + 2$.

**Using Differences as Multiples**

The fourth subject (S4) solved the problem of generalizing linear patterns using differences as multiples. A multiplication definition is that it can distinguish between multiples and non-multiples (Liljedahl, 2004). For example, the problem of a number pattern with a difference of 4, then a multiple of 4 is a number that multiplies the number 4 with a natural number, so a multiple of 4 is written $4n$. After understanding multiples, the next step is to look at the terms in the number pattern. Are the terms in the number pattern above or below the multiples? If the term is above, then the multiples are added to the rest of the term's numbers, but if the term is below, then the multiples are reduced at least the numbers in the term. For example, in the generalization of linear patterns (see Figure 1), the second multiple is 8, and the second term of the number pattern is 10. It means that the term is above a multiple of two, then the number multiples plus 2, which is $4n + 2$. Then the nth term formula from the linear pattern is $U_n = 4n + 2$. Figure 5 display the results of the work of subjects who use differences as multiples.

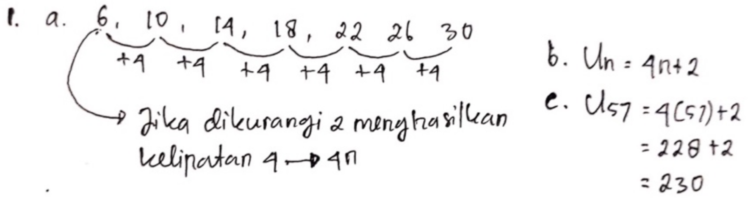


**Figure 5.** Fourth Subject Work Results

Figure 5, it clear that the subject used differences as $4n$ multiplication. Then the subject looked for the relationship of multiples with the terms in the number pattern.

So, if the numbers in the pattern numbers are reduced by 2, the result is a multiple of 4. Suppose the subject wrote the result of the linear pattern generalization $U_n = 4n + 2$ ($4n$ is obtained because the sequence is a multiple of 4 as a basis, and +2 is obtained because in rows each multiple pluses 2). The following interview excerpt shows that the subject's steps that used differences as multiples.

| | | |
|---|---|---|
| *Researcher* | : | *What do you determine the difference for?* |
| *S4* | : | *To determine the next term.* |
| *Researcher* | : | *How did you get this Generalization formula* $U_n = 4n + 2$ *?* |
| *S4* | : | *I interpret differences as multiples, sir. Rows 6, 10, 14, ... are multiples of 4 if each term is subtracted by 2. So I wrote the formula* $U_n = 4n + 2$. |

From the interview results, it clear that the steps the subject used are different to find multiples of numbers, namely: (1) the subject determined the difference from the linear pattern; (2) subjects wrote differences as multiples, i.e., b, where b is different, (3) subjects determined the compatibility of multiples with terms in a number pattern (if the terms in the number pattern are below multiples, then multiples are reduced and if the terms in the number pattern are above multiples, the multiples are added), (4) the subject wrote a linear pattern formula, namely $U_n = dn + c$ where c is positive if the term in the number pattern is above multiples, and $c$ is negative if the term in the number pattern is under a multiple, where the formula for generalizing the linear pattern is $U_n = 4n + 2$.

**Using the Difference to be Placed in the Different Column**

The fifth subject (S5) solved linear patterns by using differences to be placed in different columns. When completing using this method, the subject first used a trial and error strategy. Then the subject changed from a trial and error strategy to a different strategy. The subjects found a place to put the difference and then add the difference's first-term reduction results when using a trial and error strategy. Figure 6 displays the results of the answers to the fifth subject (S5) as follows.

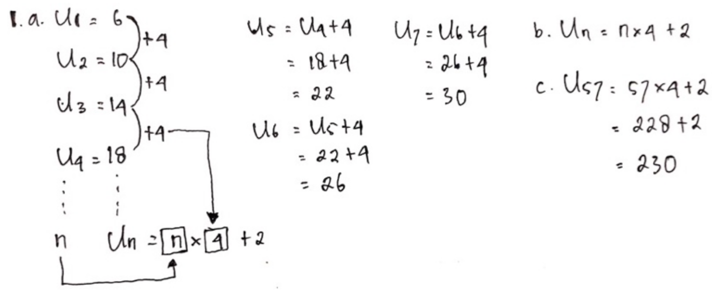


**Figure 6.** Fifth Subject Work Results

Figure 6 states that using the difference to place in a different column is done by making a column to place $n$ positions, different positions, and constant columns. The subject places the difference in the different columns, and the subject determines the number to be placed in the constant column using the first term minus the difference, namely $6 - 4 = 2$. So, the subject gets the results of the generalization of the linear pattern, namely $U_n = 4n + 2$. Using differences to place in the different columns can be seen from the following interview excerpt.

| | | |
|---|---|---|
| *Researcher* | : | *What do you determine the difference for?* |
| *S5* | : | *To determine the next term.* |
| *Researcher* | : | *How did you get this generalization formula* $U_n = 4n + 2$ ! |
| *S5* | : | *First, I tried to pack the pattern, then I found the pattern and wrote it in the form* $U_n = 4n + 2$ |
| *Researcher* | : | *Where are the n numbers from? And where are 2 and 4 from?* |
| *S5* | : | *n is the nth term if the first term means n = 1*<br>*4 was obtained from different.*<br>*2 is obtained from the first term minus the difference.* |

The interview results show that the steps of the subject in using the difference to be placed in different columns are: (1) the subject looked for different patterns of linear, (2) the subject placed the difference in different columns, (3) the subject looked for constant values by reducing the terms first by difference, and (4) the subject wrote the results of generalizing linear patterns, namely $U_n = 4n + 2$.

**Using Differences to Determine the Generalization Formula Directly**

The sixth subject (S6) solved linear patterns by using differences directly to generalize the linear pattern. Then, Figure 7 shows the results of the sixth subject's work as follows.

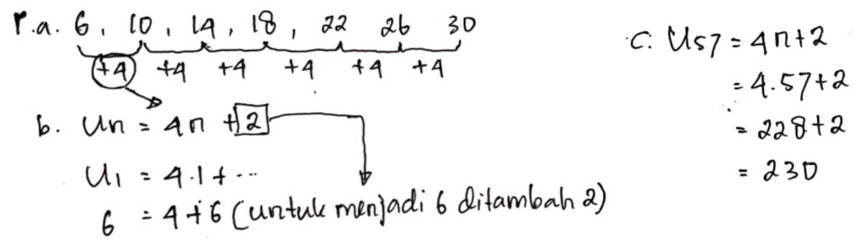

**Figure 7.** Sixth Subject Work Results

Figure 7 shows that the subject uses differences to generalize linear patterns directly, i.e., the subject looked for differences. Then the subject made the initial formula from the difference $U_n = 4n$. Subjects tested the formula using the first term, obtained $U_1 = 4(1) = 4$ (to be 6 plus 2). The subject added the initial formula with 2 so that the linear pattern generalization formula was obtained, namely $U_n = 4n + 2$. In detail, the following interview excerpt reveals the steps for using the difference to generalize linear patterns directly.

| | | |
|---|---|---|
| *Researcher* | : | *What do you determine the difference for?* |
| *S6* | : | *To determine the next term.* |
| *Researcher* | : | *How did you get this generalization formula* $U_n = 4n + 2$ *!* |
| *S6* | : | *I immediately decided.* |
| *Researcher* | : | *4n form from which? And where did you get 2?* |
| *S6* | : | *4 is different, n is the nth term (students write* $U_1 = 6$, $U_2 = 10, U_3 = 14$*). To become 6, then* $4(1) + 2 = 6$*. So I immediately wrote the formula* $U_n = 4n + 2$. |

From the interview results, it is clear that the steps of the subject used the difference to determine the formula directly, namely: (1) the subject determined the difference from the linear pattern, (2) the subject wrote the initial formula of the nth term of the difference, namely $U_n = dn$, where $d$ is a difference, (3) the subject tested the initial formula using the first term if it has a difference between the initial formula and the first term, then the first term is reduced differently, and (4) the subject wrote the formula for the linear pattern generalization by writing the initial formula added with the result of the subtraction the first term by difference is $U_n = 4n + 2$.

This study's findings reveal insights on the characteristics of different strategies and the steps used in solving linear pattern problems by analyzing the subject's work in problem-solving of linear pattern generalization and analysis of interview scripts. From the job analysis results of 40 students who solved generalizing linear patterns, 72.5% of students used different strategies. This study shows that junior high school students tend to use many different strategies. The present findings follow previous studies, showing that as many as 80% of 140 students use different strategies to solve geometric patterns or number patterns (Stacey, 1989). According to the previous studies, the strategy to generalize linear patterns primarily used numerical strategies (Becker & Rivera, 2005; Rivera & Becker, 2003).

This study also shows that the strategy to find this difference is effective for close generalizations, namely finding terms more minor than the 10$^{th}$ term of a number pattern. For distant generalizations, however, students must know the different uses for forming general rules. If students do not know the usefulness of the difference in distant generalizations, then students will be at risk of a recursive strategy. The results of a study (Hourigan & Leavy, 2015) show that if the focus is on numerical data, students will be at risk of recursive relationships, which in the end, students cannot generalize linear patterns (Becker & Rivera, 2005). This study's results contribute to developing previous studies' results, namely explaining different strategies' characteristics to solve linear pattern problems not to get caught up in recursive strategies.

The first characteristic is substituting the difference into the n$^{th}$ term formula of the arithmetic sequence, namely $U_n = a + (n-1)d$. Then, the second characteristic is by substituting the difference into the linear pattern formula, namely $U_n = dn + c$. Various mathematics books at the junior high school level provide both of these methods. The use of the n$^{th}$ term formula of arithmetic sequences and linear patterns is inseparable from school learning. The research results show that three factors: cognitive factors, task factors, and social factors, influenced linear pattern generalization strategies (Lannin et al., 2006). This study expands on previous studies' results by showing that teachers, as social factors, influence different strategies, including the most crucial ability to remember formulas. Various research results show that teacher competence affects their teaching in the classroom, affecting student achievement (Setiawan, 2020d, 2020e; Setiawan & Syaifuddin, 2020), and affects students' methods in solving problems (Setiawan et al., 2020).

The third characteristic is to use the difference as a jump number. The use of difference as a jump number followed the previous study's results, which showed that junior high school students succeeded in generalizing linear patterns by using the difference as a jump number (Setiawan et al., 2020). The subject uses a difference of 4 as a jump number, so it is written $4n$. Subjects sought the relationship $4n$ with the terms in the number pattern, so the subject succeeded in finding the linear pattern problem's nth term formula. The subject's success in using the difference as a jump number lies in looking for the jump numbers' relationship with the terms in the number pattern. It is also similar to Setiawan's research (2020c) which shows that students are confused in a trial and error strategy. Besides, they will be successful if they can see the initial formula's quantity relationship with the number pattern terms. The results also show that students are more successful in generalizing linear patterns by finding quantity relationships rather than simply finding patterns (Ellis, 2011a). Therefore, this quantitative relationship is widely used as a starting point for forming algebraic reasoning (Kaput, 2008; Kaput, Blanton, & Moreno, 2008).

The fourth characteristic is to use the difference as multiples. This study's results follow previous studies' results, which showed that students succeeded in generalizing linear patterns by using differences as multiples (Liljedahl, 2004). This study expands on the results of previous studies by explaining how students use differences as multiples. There are two primary keys to successfully generalizing linear patterns by using differences as multiples, namely: (1) understanding multiples and (2) paying attention to the terms in a number pattern above or below a multiple. If the term is above, then the multiples are added to the rest of the term's numbers. If the term is below, then the multiples are reduced by at least the term's numbers. For example, in the number pattern 6, 10, 14, 18,… the difference is 4. Each term is considered a multiple of 4. To get the first term is $4(1)+2=6$, the second term is $4(2)+2=10$, and so on. Thus we get the formula for the nth term of the number pattern 6, 10, 14, 18, ... is $4n+2$.

The fifth characteristic is to place differences in different columns, and the sixth characteristic is to use differences to determine the formula for generalizing linear patterns directly. This study results from recent studies that contribute to adding to the different strategic approaches that have been found previously (Lannin et al., 2006; Liljedahl, 2004; Setiawan et al., 2020; Setiawan, 2020c). Placing the difference in this different column (see Figure 8) is a more effective way of generalizing linear patterns. Figure 8 shows that the effectiveness of placing the difference in this different column is enough to place the difference and the results of the first term minus the difference in the column provided.

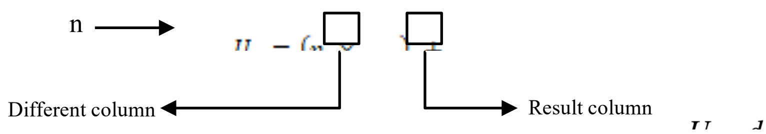


**Figure 8.** Different columns and results column $U_1 - d$

The characteristics of different strategies in generalizing this linear pattern arise due to students' creativity in finding the relationship between the quantities in the number pattern. The definition relationship between quantities is the conception of three quantities, two of which determine the third by quantitative operations (Kabael & Akin, 2018; Nunes, Bryant, Evans, Bell, & Barros, 2012; Tallman & Frank, 2020). For example, the relationship between 2, 4, and 10 is $4(2)+2=10$. The results of this study explain how subjects use this quantity relationship in generalizing linear patterns. Subjects who use different strategies as jump numbers say that the first term is the jump number 4 once, the second term is the jump number 4 twice, the third term is the jump number 4 three times, and so on. Then the students look for the quantity relationship between the jump numbers and the known terms (for example, the relationship 4 to 6 is $4(1)+2=6$). Subjects who used the difference as a multiple say that difference (i.e., 4) is a multiple ($4n$). Then the subject looks for the relationship between the multiple numbers ($4n$) and the known term. Likewise, subjects who generalized directly appear to find the relationship between differences, known terms, and the value of *n*. From this quantity relationship, the students succeeded in generalizing the linear pattern by finding the general formula.

The results of this study have implications for learning linear patterns using different strategies. Through these different characteristics of different strategies, students will know the usefulness of the differences in generalizing linear patterns. The researcher recommends that the teachers learn the linear pattern using various characteristics of different strategies. The reason is that students have different understandings, so students will choose the characteristics of the different strategies which students find easiest to use. The study of Yilmaz and Argun (2018) said that when someone thinks or talks about mathematical objects or processes, they will connect with the perception in his mind. Furthermore, the learning of linear patterns should emphasize understanding concepts rather than just memorizing because understanding concepts is essential knowledge that forms the basis of procedural knowledge (Setiawan, 2020a, 2020c; Setiawan & Mustangin, 2020; Setiawan & Syaifuddin, 2020).

## CONCLUSION

Although this research is limited to six research subjects, this study contributed to developing a linear pattern generalization strategy to characterize different strategies to generalize linear patterns. Characteristics of different strategies in generalizing linear patterns consist of: (1) using the difference to substitute into the nth term formula of arithmetic sequence; (2) using the difference to be substituted into a linear pattern formula; (3) using the difference as a jump number; (4) using differences as multiples; (5) using differences to be placed in different columns; and (6) using differences to determine the formula for generalizing linear patterns directly. This different strategy's characteristics arise because students are creative in using differences by finding the quantitative relationship between differences and the number pattern's known terms. Through this study's results, the objective is to overcome students' ignorance in using difference, so students can solve the problem of generalizing linear patterns correctly. This research is only limited to the different strategies in generalizing linear patterns. Further research can identify students' mistakes in using specific generalization strategies and provide alternative solutions.

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