Learning Divisions of Fractions through Sprint Running Pictures

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Abstract
This study aims to produce Learning Trajectory (LT) that can help students understand the division of fractions in grade 5 elementary schools. This study was conducted on 30 students in class V.e MIN 2 Palembang, South Sumatra Province, Indonesia. This study uses Design Research method with three research stages, namely: 1) Preliminary Design/preparing for an experiment which is the stage of designing Hyphotecal Learning Trajectory (HLT) and instruments needed, 2) Design Experiment namely HLT test phase consisting of Pilot Experiment and Teaching Experiment and 3) Retrospective Analysis. Instruments for data collections are video recordings, documentation, and interviews, observation sheet, and pre-test and post-test. Hypothetical Learning Trajectory (HLT) of learning division of fractions has been designed into Learning Trajectory (LT) so it can function as a Local Instructional Theory (LIT) which can be developed to provide the materials of the division of fractions for students. The results of this study indicate that the trajectory of learning produced can help students understand the division of fractions. Therefore, it will be able to contribute positively to the various parties who have an interest in the education such as teachers, students, schools and the government. Furthermore, it is also can be used as a reference in the teaching and learning and developing materials of teaching mathematics.

Keywords: Divisions of Fractions, HLT, Sprint, model bar, Design Research

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Received July 7, 2017; Revised August 28, 2017; Accepted October 10, 2017
Introduction

One of the fundamental concepts in mathematics is fractions. Sukajati (2008) stated that "fractions is one of the concepts that are considered difficult to be understood by students and even teachers". From various studies that have been done, it is noted that the division of fractions is one of the toughest material in arithmetic (Greg & Greg, 2007; Zaleta, 2008; Coughlin, 2010; Yukans, 2012).

One of the causes of errors students made in solving division of fractions is the error in the use of the algorithm. This is caused by students' misunderstanding of the concept of division of fractions. Ekawati (2008), stated that "the reason why the fractions are one of the materials that are difficult in mathematics because usually mathematics in primary schools are based on things that are abstract". In teaching fractions, teachers tend to immediately focus on procedures for using algorithms only. They do not start from the real context for students while many things can be used as the real context to study mathematics (Ulya, 2010; Diah, 2013).

Some research has been done to build an understanding of division of fractions, among others are Bulgar (2009), using the context of the ribbon in the 4th grade for the division of fractions related to Measurement Division. Yukans (2012), conducting research at MIN 2 Palembang to develop students' understanding of studying the inverse relationship between multiplication operations with divisions of fractions operation on Measurement Division and Positive Division, where students explore issues of measurement division and partition division by context ribbon. Amiati (2013) did research by conducting activities on measurements and create a partition to solve multiplication and division problems. Zaleta (2008) used the contextual situation and some real objects for students to learn their first fractions. The main focus of her research is the calculation of division and only showed how to resolve the problem informally. Greg & Greg (2007) separated division problem into two main problems, measurement, and distribution of the partition.

On that note, the researchers will apply different techniques to teach division of fractions, namely by using a model of the bar to the context of the World Sprint run record.

The purpose of this research is to produce a learning trajectory that can help students understand the division of fractions through the context of the World Sprint run record. The major problems of this study are how HLT can help students understand the material division of fractions by using a bar context Sprint World record run? The benefits of research can be categorized in two ways namely practically benefit and theoretically benefit. Theoretically, this study contributes to grounded instructional theory in learning division of fractions, namely the use of Van de Walle theory which divides by measuring and dividing by partitioning, in this case using partition bars. Practically, this study provides an overview for teachers and researchers on how to design a study that emphasizes understanding, especially on the topic of the division of fractions.

Methods

This study involves 30 of 5rd grade students of State elementary school MIN 2 Palembang that consisted ranging ages from 10 to 11 years. This study also involves a 5rd grade classroom teacher of State elementary school MIN 2 Palembang, Indonesia.

This study uses research methods research design which is a cyclical process of the thought experiment and instruction experiment (Gravemeijer, 1994; Sembiring, Hoogland dan Dolk, 2010). The cyclic process (repeated) is a thought experiment to an experiment then of learning in the form of a diagram with illustrations about the experimental idea of Gravemeijer and Cobb (in Akker, 2006) shown in the image below:

![Figure 1. Reflexive Relationship between Theory and Experiment](image-url)
Instructional design is done by designing and through three stages. The stages in this study are as follows:

1) Stage I: Preliminary Design/ preparing for experiment
   a. At this stage, the researchers conducted a review of the literature regarding the division of fractions, bar theory, and Realistic Mathematics Education followed by a discussion between researchers and teachers Mrs. Desi S, Pd about classroom conditions, research purposes (Instrument), scheduling and conducting the study with the teacher.
   b. The researchers designed learning and hypothetical learning trajectory. The conjecture of local instructional theory was formulated consisting of learning objectives, learning activities, and tools to assist the learning process. Conjecture is intended as a guide to anticipate student’s emerging strategy and thrive on learning activities. Conjecture is dynamic so it can be adjusted and revised during the actual process of learning (teaching experiment).

2) Stage II: The design experiment.
   The stage 2, the design experiment, consisted of two cycles of namely cycle 1 (pilot experiment) and cycle 2 (teaching experiment). In cycle 1 (Pilot Experiment) three students were selected for heterogeneous capabilities (1 high ability students, 1 average ability student, and 1 low ability students), on this cycle of researchers acted as teachers. Results from this cycle were used to revise the initial version of the HLT, then used for learning in the second cycle (teaching experiment). In the second cycle, the students are taught by their own teacher and researchers acted as an observer to the learning activities.

3) Stage III: Retrospective Analysis
   After the experiment, the data obtained from learning activities in the classroom are then analyzed and the results are used to plan activities or to develop the design in the following learning activities. The purpose of the Retrospective Analysis is to develop local instructional theory. At this stage, HLT was compared with actual student learning and obtained answers from the formulation of the problem.

Data collection is done through a few things include: video and photo record, Observation, interviews, Documentation. Design Research is a qualitative research method, the techniques of data analysis in this study conducted qualitatively based on the results of data collection that has been done. The role of researcher in the learning activity are to stand by, ask the students some additional questions, to observe the learning activity, to coordinate the activity, and to make last-minute change to the activity that is necessary for providing relevant information for research (Risma, Ratu & Hartono, 2013; Van Nes & Van Eerde, 2010). It has two main advantages which support the aim of this study. First, it is allowed to make interventions in order to encourage the students to elaborate on their statement. Second, the clinical interview also provides a continual interaction between inference and observation (Sylvana, Frans van Galen, Zulkardi & Darmawijoyo, 2014).

Result

This study used Design Research methods by following the three stages that exist namely Preliminary Design/ preparing for the experiment, the Design Experiment, and Retrospective Analysis. This study was designed to produce a learning trajectory/learning Trajectory (LT) in using a fractions division bar with the context of the World Sprint Record run and implemented in MIN 2 Palembang involving three students at Pilot Experiment and 30 students at the Teaching Experiment. The results obtained in this study can be described as follows:

Stage 1 Result: Preliminary Design/Preparing for Experiment

At this stage, the researchers designed the Hypothetical Learning Trajectory (HLT) on the division of fractions by using a bar context Sprint world record run as a learning trajectory for the division of fractions. The context of Sprint world record run was selected because the position of each runner presented the bar, but previously the researchers reviewed the literature on the division of fractions, and what concept that can be used as teaching material in the division of fractions (such as: ratio, simplifying fractions, equivalent fractions, operations of addition and subtraction of fractions, and multiplication), World record run Sprint context, learning division of fractions in the curriculum, PMRI approach, and design research were used as research methods. Furthermore, researchers discussed it together with the mathematics teacher who will be a model of the HLT that have been designed and made assumptions about students’ response to activities designed. In addition to HLT, researchers also
designed the lesson plans, teacher’s manual and assessment instruments. The HLT has been designed is as follows:

![Hypothetical Learning Trajectory (HLT)](image)

Figure 2. Hypothetical Learning Trajectory (HLT)

Student's thinking strategy based on Hypothetical Learning Trajectory (HLT) which has been designed is that the first student will create partition on the bar, then determine the result of fractional division by using bar, lastly find the concept of fractional division. From the designed HLT, three activities were generated, ie activity 1 was aimed at creating partitions on the bar, activity 2 was aimed at determining the division of dividing by using bar, and activity 3 was aimed to find the fractional division concept.

Stage 2 Result: The design experiment

At this stage, there are two cycles. Cycle 1, known as Pilot Experiment and cycle 2, known as the Teaching Experiment. The results of the second cycle can be described as follows: In Cycle 1 three students have been categorized as having good ability, average and low. All three students were given activity sheets that have been designed and validated by their teachers at stage 1. Then, the students’ work were observed and they were interviewed to see their thought processes in resolving the problems that exist in each activity. The results of students’ work can be described as follows: In activity 1, students were given a picture of the world record for the fastest runners, then they were asked to complete the table provided based on the information on the picture and they were asked to create a bar based on the table that have been completed. The results of the students’ answers can be described as follows:

![World Sprint Record](image)

Figure 3. World Sprint Record
From the figure 4.a, students completed a table provided by the information contained in figure 3, then the students made comparisons into the form of fractions and simplify them. Furthermore, on the figure 4.b students were able to make bar illustrating the position of athletes in the last 10 meters of the track based on illustrations 3.To see the students’ thought process in completing activity 1, the researchers conducted interviews to students. The results of the interview can be seen from the following transcript:

**(R: Researcher, S: Student)**

Transcript 1.

R: “Why is Justin Gatlin part \(\frac{9}{10}\)?”

S: “As seen from the image, Justin is situated on the box 9 from a total of 10 boxes”

R: “how can you make \(\frac{9}{10}\) into \(\frac{4}{5}\)?”

S: “I simplify it, 8:2 = 4 and 10:2 = 5”

R: “why should it be divided by 2 and not the other numbers?”

S: “well it should be divided by the same factor”

R: “ok, can you make the bar from the results of this table?”

S: “sure”

R: “yes please try”
From the transcript 1 above it is known that the student has understood how determine the position of each athlete in the final 10 meters track, student was also able to simplify fractions by dividing the numerator and denominator of the fractions by the same factor. Then the student determined the position of the athletes using the illustration bar in accordance with table he had fill out.

In the second activity, students were asked to determine the position of the athlete in the final 10 meters track against the positions of the other athletes. The purpose from this activity was that students can perform fractions division by using partition bars, the results are as follows.

In question Number 1 students were asked to determine how many part that Jesse Own has to cover to catch up with Armin, this is necessary because it served as the basis for solving the division of fractions by fractions using a bar. The result is

\[
\frac{6}{10} : \frac{3}{10} = 2
\]

Furthermore, the knowledge students gained from previous activities (activity 1 and 2) will be used to complete the activity 3. The students’ work in activity 3 can be described as follows:

In Question activities 3, students were asked to solve \( \frac{6}{10} : \frac{3}{10} \), using their knowledge from previous activity, the student's work is

\[
\frac{6}{10} : \frac{3}{10} = 2
\]
From figure 6, it can be seen that students can solve $\frac{6}{10} \div \frac{3}{10}$ by using the partition bar. Their strategy is dividing the bar into 10 parts, and this is accomplished by the students with reference to previous activity. Furthermore, students will first determine the location of $\frac{6}{10} \div \frac{3}{10}$ by counting box that they have made at the bar. Next, student determine the position of $\frac{3}{10}$, then how many boxes they need to shade to cover $\frac{6}{10}$. Therefore, they found out that there were 2 parts of $\frac{3}{10}$ needed to cover $\frac{6}{10} \div \frac{3}{10} = 2$.

Retrospective analysis of cycle 1

From the students’ work, in general they can be said to understand the activities designed well enough since they used information obtained from previous activities. Therefore, students were able to find the results of the division of fractions by partition bar provided as a guide. Then based on the results done of problems that exist in the activity of 1, 2 and 3, students could conclude the general form of the division of fractions. There was no difficulty for students to solve any questions presented in activity 1, activity 2, 3 and corresponding activities with the HLT and no revision of HLT designed.

In the second cycle the study was conducted on 30 students of class Ve at which researcher acted as observer. The students’ work were observed and they were interviewed to see their thought processes in solving given problems in each activity. The results of students’ work can be described as follows:

At this stage, same as the pilot stage of experiments for activity 1, students were given a picture of the fastest sprinter of the world record, then they were asked to complete the table provided based on the information on the picture, and then after LAS was distributed and before students discussed in groups, students work independently first. After the students completed all questions in activity 1, they continued discussing with the group of their friends to get the best answer.

While students discuss with the group of their friends, observers asked the answers obtained by the students. Furthermore, students presented their work in front of the class. Here are the results of interviews with student on activity 1.

Transcript 2.

R: where did you get this? (pointing at student’s bar)
S: from this! (pointing at World’ Sprint Record)

Each box is filled with number 1
R: How many boxes are there?
S: 10
R: how many boxes is shaded for Usain Bolt?
S: 10 boxes
R: What about Justin?
S: 9 boxes
R: what does that mean?
S: it’s 9/10 part for Justin

From the interview above, students were able to determine which parts of each of the runners in the last 10 meters on the track by making illustrations on the bar. Here are the work of one group of activity 1.
The results of activity 1 was no different from results obtained in cycle 1 and will be used as additional information for accomplishing activity. The teacher asked one of the groups to present the results of the group discussions, students’ presentations could be seen from the image below:

Further information obtained of activity 1 will be used to complete the second activity, in this stage learning activities have led to how to find the division of fractions, but the questions still to be situational. Just like when students completed 1 activity, in activity 2 students worked individually first and then discussed with group of their friends, the following picture when students discuss with the group of their friends:
While students discuss with the group of their friends, observers asked the answers obtained by the students. Furthermore, students presented their work in front of the class. Here are the results of interviews with student on activity 2.

**Transcript. 3**

R: *How did you get this \( \frac{3}{10} \)?* (pointing at student’s answer)
S: *from 3 boxes*
R: *which ones?*
S: *1,2,3,* (while pointing and counting boxes)
R: *how can you get \( \frac{3}{10} \)?*
S: *‘cause these are 3 boxes, the total was 10*
R: *so how many is Jesse’s part?*
S: \( \frac{3}{10} \)
R: *What about Armin’s?*
S: \( \frac{6}{10} \)
R: *so how many Jesse’s part needed to cover Armin’s part?*
S: *there are 2*

From the interview above, it is known that the students have understood the use of the bar. The work of students in the second activity can be described as follows:

**Figure 10. Students’ Answer to Question Activity 2**

The results of activity 2 was no different from results obtained in cycle 2 and will be used as additional information for accomplishing activity. The teacher asked one of the groups to present the results of the group discussions, students’ presentations could be seen from the figure below:
In the third activity, students are asked to look for the division of fractions by fractions. In this activity, students were given a picture of the fastest sprinter of the world record, then they were asked to complete the table provided based on the information on the picture, and then after LAS was distributed and before students discussed in groups, students work independently first. After the students completed all questions in activity 3, they continued discussing with the group of their friends to get the best answer.

While students discuss with the group of their friends, observers asked the answers obtained by the students. Furthermore, students presented their work in front of the class. Here are the results of interviews with student on activity 3.

**Transcript 4**

O: How did you get this $\frac{6}{10}$?
S: We have the bar here divided into 10 parts, there are 6 shaded boxes on the bar so $\frac{6}{10}$.

O: What about this $\frac{3}{10}$?
S: I divide $\frac{6}{10}$ boxes into two, so each gets 3 boxes, $\frac{3}{10}$ and $\frac{3}{10}$

O: How many $\frac{3}{10}$ do you need to cover $\frac{6}{10}$?
S: There are 2 parts of $\frac{3}{10}$ to cover $\frac{36}{10}$.

O: Which means?
S: $\frac{6}{10} : \frac{3}{10} = 2$

From the interview above, it is clear that students were able to solve the division of fractions by fractions using a bar. The results obtained in cycle 2 was not different from those obtained in cycle 1, where students used algebra immediately. Similarly in cycle 2, students were actually capable of doing algebra using a direct way, but when asked how they got the answer, students were confused since they said they have learned it in tutoring, but not taught about the origin of the use of the algorithm. Here are students work in accomplishing activity 3.
Retrospective Analysis of Teaching Experiment.

From the work of the students, in general the student could understand the activities designed well enough since they were using information obtained from the previous activities. Therefore, students were able to find the results of division of fractions by partition bar provided as a guide. Then based on the results done of problems that exist in the activity of 1, 2, and 3, students could conclude the general form of the division of fractions. There was no difficulty for students to solve any questions presented in activity 1, activity 2, 3 and corresponding activities with the HLT.

Discussion

The design of learning trajectory designed and conducted by researchers to understand the division of fractions using the theory of Van de Walle is dividing by using partition bar covering three learning activities that have been performed on the process of learning by students. By using PMRI approach, a series of activities to generate a trajectory study was carried out in every cycle: cycle 1 and cycle 2 Pilot Experiment and Teaching Experiment.

This learning is based on the implementation of PMRI in designing each learning activity that shows the characteristics PMRI to be based at each activity. Five characteristics of PMRI which is the adoption of RME according to Zulkardi (2002) in relation in this learning are: 1). Using contextual problems, 2). Using a model, which aims to connect between something concrete to the abstract or the informal level towards the more formal level (Freudental, 1991). According to Gravemeijer (1994), there are 4 level in RME, which are situational, model of, model for, and formal. 3). The utilization of the contribution of students. 4). Interactivity, 5). Connection to other learning topics. In general, from the first to the last activity, the students have done a whole set of measurement activity and create a partition in PMRI by using bar.

In terms of conjectures, which is designed in this study to anticipate the strategy / reasoning of students, most of the conjectures has been prepared in accordance with the strategy of student thinking. Thus the findings in this study is an integral part of developing local instructional theory in this case PMRI approach in learning division of fractions.

Conclusion

Based on the results and the discussion above, it can be concluded that:

1. Hypothetical Learning trajectory that has been implemented in this study has become the Learning Trajectory that can help improve students’ understanding of lesson.
2. The Learning Trajectory implemented in this study is one of a positive contribution to develop the Local Instructional Theory (LIT) in learning division of fractions.

In general, the HLT designed has become the Learning Trajectory that helps students learn in performing operations of division of fractions. Therefore, researchers suggest:

1. To teachers.
   It is advisable to implement and develop the Learning Trajectory of divisions of fractions in this research results in their own learning activities. Teachers are also expected to be actively involved in designing the lesson to other materials using the suggested lesson to the curriculum that applies.
2. To students.
   Students should be more active and brave in expressing their opinions, thoughts, and ideas so that they will be able to get the ultimate the learning process in participating in learning activities.

3. To other researchers
   This study only focused on teaching divisions of fractions, so it is expected in the future other researchers can develop this research not only for teaching division of fractions, but also for other materials.

Acknowledgement

This research can be done with grant aid from the ministry of higher education, so the Researcher would like to thank the Director General of Higher Education and the head of the Faculty of Teacher Training of Sriwijaya University who has helped facilitate this research.

References

Zulkardi. “Developing A Learning Environment on Realistik Mathematics Education for Indonesian Student Teachers,” in Doctoral thesis of Twente University is not published, ed Enschede: Twente University, 2002.